Journal of Physics: Conference Series

Finite Elements and Finite Differences in some differential equations of second linear order with GeoGebra

To cite this article: Jorge Olivares Funes and Elvis R. Valero Kari 2021 *J. Phys.: Conf. Ser.* **1730** 012068

Finite Elements and Finite Differences in some differential equations of second linear order with GeoGebra

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Abstract. In this paper we will show the numerical solutions of some linear second order differential equations through the finite difference methods with Lagrange interpolation and finite elements, using GeoGebra software. These numerical solutions mediated by GeoGebra are visualized with dynamic applets, which were built to analyze the absolute error produced by the corresponding solution and the approximation and also as an interactive teaching support material for the course of Numerical Calculation for Engineering and Science students .

1. Introduction

Many researchers currently recommend the use of educational and interactive software as stated by the NCTM [2] which are cited in [1] and tell us that software is an alternative that links technology with mathematical tools as an essential resource with the object of helping students learn, make sense of mathematical ideas, reason mathematically and communicate their mathematical thinking. According to Flores [4] cited in [3], the GeoGebra software has a simple interface and a great variety of geometric and algebraic tools that allow a large number of constructions to be achieved and achieved. Based on the previous statements it is that animated Geogebra applets of numerical approximations of the solutions of the differential equations of linear second order were made under the method of finite differences with interpolation of Lagrange and finite elements, for the numerical calculation course of the University of Antofagasta during the year 2019 and available at https://www.geogebra.org/m/n9qkpmst.

This work is a continuation and extension of the paper, "Resolving non-homogeneous linear differential equations using the undetermined method coefficients and variation of parameters by means of GeoGebra" [5]

2. Finite Difference and Finite Element Applets

Of the applets already built, two will be shown especially for the same differential equation, where you can visualize the approximation of the solution with finite differences with interpolation of

Lagrange and finite elements. It should be noted that both methods worked in the interval $[0, a]$ with $h = \frac{a}{3}$

Be

$$\frac{d^2y}{dx^2} - y + f(x) = 0,\ y(0) = 0, y(a) = 0 \tag{1}$$

Suppose that $0 = x_0 < x_1 < x_2 < x_3 = a$ represents a partition of the interval $[0, a], x_1 = \frac{a}{3}, x_2 = \frac{2a}{3}$.

where for finite differences

$$x_0 = 0, y(0) = y_0 = 0; x_1 = \frac{a}{3}, y_1 \text{ unknown}; x_2 = \frac{2a}{3}, y_2, \text{ unknown}; x_3 = a, y(a) = y_3 = 0$$

to get the value of y_1 and y_2 in terms of a we solve

$$\begin{aligned} y_2 + \left(-2 - \tfrac{a^2}{9}\right) y_1 + h^2 f(x_1) = 0 \\ y_1 + \left(-2 - \tfrac{a^2}{9}\right) y_2 + h^2 f(x_2) = 0 \end{aligned}$$

Example 1

$$\frac{d^2y}{dx^2} - y + \frac{x}{a} = 0, y(0) = y(a) = 0, a > 0$$

Whose solution is

$$y(x) = \frac{e^{a-x} - e^{a+x}}{e^{2a} - 1} + \frac{x}{a},\ x \in [0, a].$$

Be $h = \frac{a}{3}$*, with* $y_0 = y_3 = 0$ *y* $f(x) = \frac{x}{a}$.

The value obtained for y_1 *and* y_2*, in terms of a is*

$$y_1 = \frac{a^2(a^2 + 36)}{3(a^4 + 36a^2 + 243)}$$

$$y_2 = \frac{a^2(2a^2 + 45)}{3(a^4 + 36a^2 + 243)}$$

Then, this determines the Lagrange polynomial for the points $(0, 0), (a/3, y_1), (2a/3, y_2), (a, 0)$.

In the figure 1 a varies betwen 0.1 *and* 10*, for example for* $a = 3$ *the Lagrange polynomial with red is*

$$p_3(x) = -0.04167 + 0.0625x^2 + 0.1875x,$$

and the absolute error is 0.02109 at $x = 2.52265$

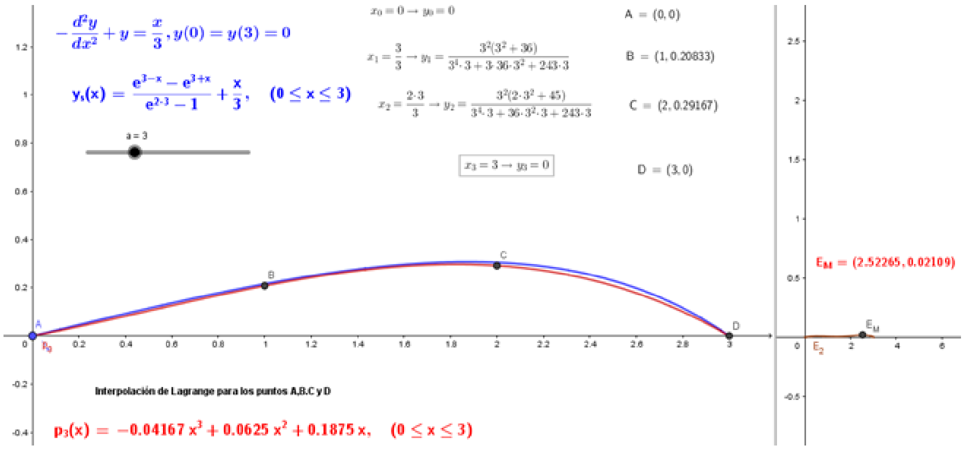

Figure 1. The solution is blue and the approximation with finite difference is red

Be $-\frac{d^2y}{dx^2}+y=f(x), y(0)=0, y(a)=0$. *Suppose that* $0=x_0<x_1<x_2<x_3=a$ *represents a partition of the interval* $[0,a], x_1=\frac{a}{3}, x_2=\frac{2a}{3}$.

An approximation for this finite element solution is given by
$z(x)=\alpha\Phi_1(x)+\alpha_2\Phi_2(x)$ *with*

$$\Phi_1(x)=\begin{cases}\frac{3x}{a}, & x\in[0,a/3]\\ 2-\frac{3x}{a}, & x\in[a/3,2a/3]\\ 0, & x\in[2a/3,a]\end{cases} \quad \Phi_2(x)=\begin{cases}\frac{3x}{a}-1, & x\in[a/3,2a/3]\\ 3-\frac{3x}{a}, & x\in[2a/3,a]\\ 0, & x\in[0,a/3]\end{cases}$$

And the values α_1 *and* α_2 *are obtained by solving the system*

$$\left(\frac{2a}{9}+\frac{6}{a}\right)\alpha_1+\left(\frac{a}{18}-\frac{3}{a}\right)\alpha_2=\int_0^a f(x)\Phi_1(x)dx$$

$$\left(\frac{a}{18}-\frac{3}{a}\right)\alpha_1+\left(\frac{2a}{9}-\frac{6}{a}\right)\alpha_2=\int_0^a f(x)\Phi_2(x)dx$$

Example 2

$$\frac{d^2y}{dx^2}-y=-\frac{x}{a}, y(0)=y(a)=0.$$

Whose solution is $y(x)=\frac{e^{a-x}-e^{a+x}}{e^{2a}-1}+\frac{x}{a}, x\in[0,a]$.
Be $y_0=y_3=0$ *and* $f(x)=-\frac{x}{a}$.
The value obtainet α_1 *and* α_2*, in terms of* a *is*

$$\alpha_1=\frac{4a^2(a^2+108)}{15a^4+972a^2+8748}$$
$$\alpha_2=\frac{2a^2(a^2+270)}{15a^4+972a^2+8748}$$

In figure 2 it can be observed, for example, that $a=3$ *the error absolute obtained is* 0.18888 *for* $x=2$

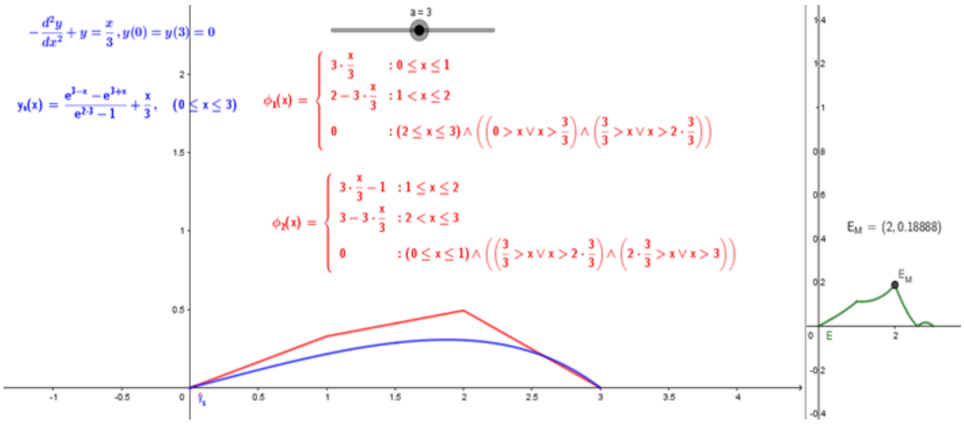

Figure 2. The solution is blue and the approximation with finite difference is red

3. Conclusion

We see that through these GeoGebra applets applied to the same differential equation, the finite difference method with Lagrange interpolation is more advantageous than Finite element, Now the next step is if this same advantage is maintained but with greater elements in the interval partition left $[0, a]$.

Also note that in both methods as a assumes large values, the absolute error tends to be greater, so it is more advisable to work for adequate and small a values.

References

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