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Galarkin Method with GeoGebra in Differential Equations

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Galarkin Method with GeoGebra in Differential Equations

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Abstract. Let us consider

$$\frac{d^2y}{dx^2} + y = Q(x,a), y(0) = y(1) = 0, x, a \in (0,1).$$

In the following paper, various differential equations will be displayed, which willbe solved using Galerkin's numericla method and where formal solutions and their numerical approximations can be seen with GeoGebra animated Apptles.

Keywords: Numerical methods; Differential Equations; Geogebra; Modeling in Education

1. Introduction

Today, the use of numerical methods for the resolution of differential equations for research or teaching is very present in science and engineering, as mentioned in ref. [5].

This is why this paper contributes and extends the works [1-4], and it is expected to help researchers or estudents.

Some of the differential equations presented below were used in the Courses of Differential Equations and Numerical Calculation for Engineering Careers at the Universidad de Antofagasta, Chile.

2. Galarkin Apptles With GeoGebra

The examples below are available at the following link https://www.geogebra.org/m/vnjb2m6v

Example 1

Let the equation

$$\frac{d^2y}{dx^2} + y = -ax, y(0) = y(1) = 0 \tag{1}$$

whose blue solution is $y - a \cdot cosec(1)\sin(x) - ax$. The Galarkin's approach in red is

$$U_2 = \frac{71a}{369}x(1-x) + \frac{7a}{41}x^2(1-x).$$

See Figure 1, where a ranges from 0.1 to 1.

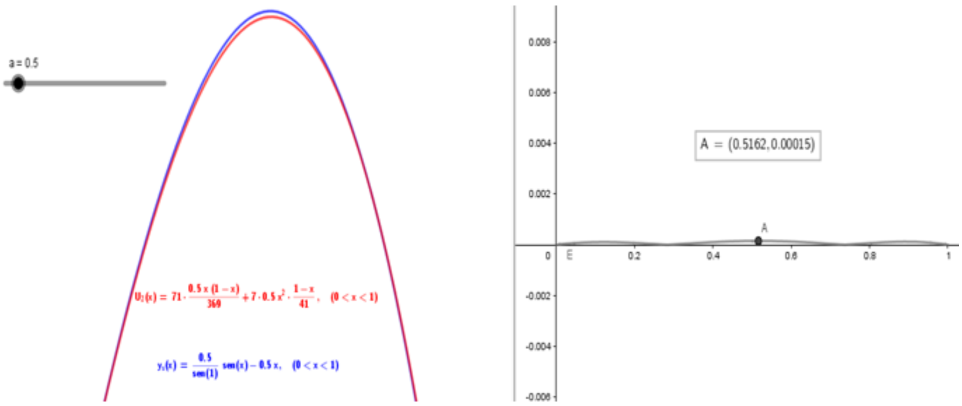

Figure 1.

In this case for $a = 0.5$, the maximum absolute error is 0.00015.

Example 2

Let the equation

$$\frac{d^2y}{dx^2} + y = -a, y(0) = y(1) = 0 \tag{2}$$

whose blue solution is $y - a \cdot (\cos x \tan(1/2)\sin(x) - 1)$. The Galarkin's approach in red is $U_2 = \frac{5a}{9}x(1-x)$. See Figure 2 where a varies between 0.1 and 1.

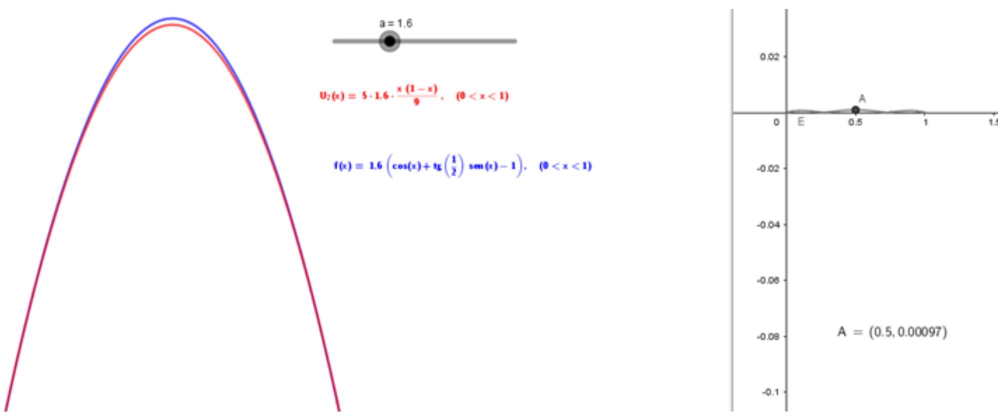

Figure 2.

In this case for $a = 1.6$, the maximun absolute error is 0.00097.

Example 3

Let the equation

$$\frac{d^2y}{dx^2} - y = a^2x, y(0) - y(1) = 0 \tag{3}$$

whose solution in blue is $y - a^2e^{-x}(xe^x - xe^{x+2} + e^{2x+1} - e)\frac{1}{e^2-1}$. The Galarkin's approach in red is $U_2 = 5a^a(119a^2 - 252a + 116)\frac{x(1-x)}{77a^2-8a+404} - 35a^2(5)a^2 + 14a - 8\frac{x^2(1-x)}{77a^2-8a+404}$. See Figure 3, where a ranges from 0.1 to 1.

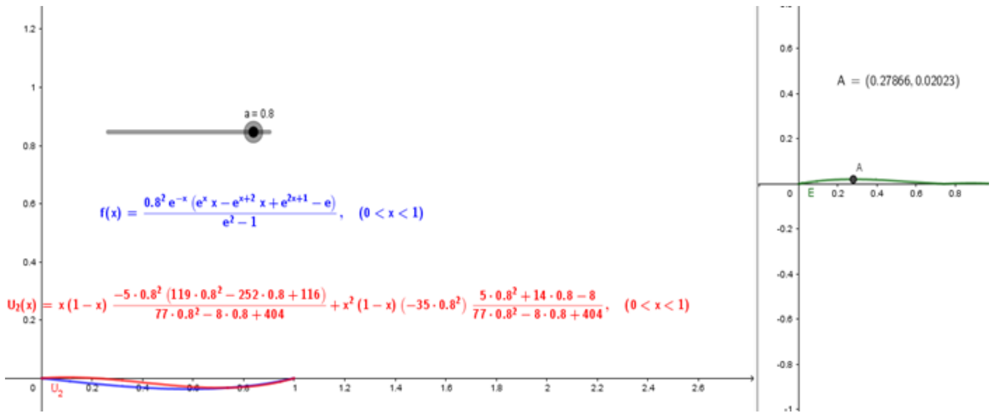

Figure 3.

In this case for $a = 0.8$, the maximum absolute error is 0.02023.

Example 4

Let the equation

$$\frac{d^2y}{dx^2} - y = a, y(0) - y(1) = 0 \tag{4}$$

whose solution in blue is $y = ae^{-x}(e^x - 1)(e^x - e)\frac{1}{1+e}$. The Galarkin's approach in red is $U_2 = -5a(7a + 36)\frac{x(1-x)}{77a^2-8a+404} + 35a(a-1)\frac{x^2(1-x)}{77a^2-8a+404}$. See Figure 4, where a ranges from -3 to 3.

In this case for $a = 0.7$, the maximum absolute error is 0.0051.

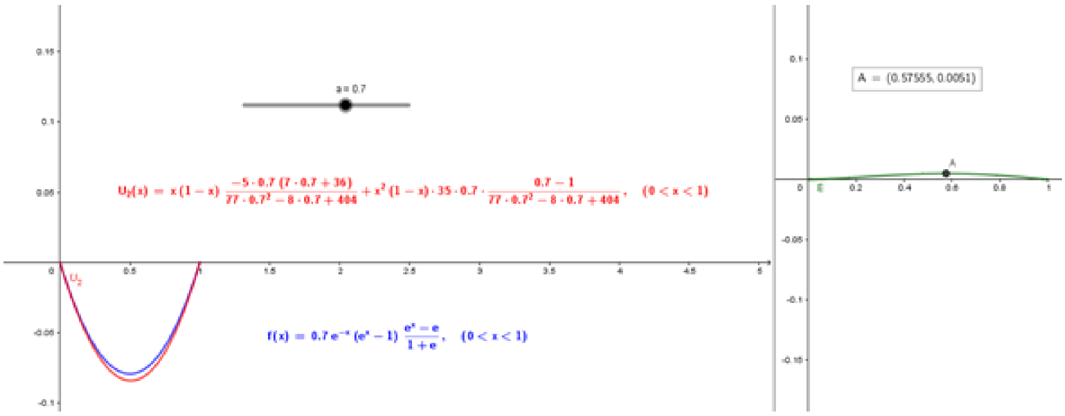

Figure 4.

3. Conclusion

The main purpose of this work has been to motivate the various researchers, teachers and students of higher education in the use of Geogebra applets in teachin and learning the Galarkin method.

These applets are also expected to be the way for other numerical analysis content and topics

4. References

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