Journal of Physics: Conference Series

PAPER • OPEN ACCESS

Numerical solution of Hermite differential equation using the spline method of order 1 with GeoGebra

To cite this article: Jorge Olivares Funes and Elvis Valero Kari 2021 *J. Phys.: Conf. Ser.* **2090** 012091

Journal of Physics: Conference Series **2090** (2021) 012091 doi:10.1088/1742-6596/2090/1/012091

Numerical solution of Hermite differential equation using the spline method of order 1 with GeoGebra

Jorge Olivares Funes
Universidad de Antofagasta
E-mail: jorge.olivares@uantof.cl

Elvis Valero kari
Universidad Mayor de San Andres; Universidad de TarapacÁ
E-mail: ervalero@fcpn.edu.bo; evalero@academicos.uta.cl

Abstract. In this paper we will show the visualization of the approximations that can be obtained by means of the order 1 spline method for Hermite differential equations with well-interactive examples of GeoGebra applets.

Here, we will dedicate ourselves to publisize, the great benefit that can be obtained, in the process of generating new mathematical knowledge for learning and teaching the numerical solutions of differential equations.

Keywords: GeoGebra, Spline, Hermite equation.

1. Introduction

Various authors consider GeoGebra software as a program of great help for learning mathematics, as mentioned in ref. [1] in a textual way "Dynamic geometry software like GeoGebra is an excellent resource that allows us to model and simulate different mathematical problems, different topics Algebra subjects, Geometry and Trigonometry, Analytical Geometry and Calculation"

In our case, GeoGebra applts to visualize and see some approximations of Hermite differential equations using the order 1 spline method.

All these applets were designed and built to support teaching material for various online courses of differential equations at the University of Antofagasta during the year 2020.

2. Spline of order 1 in the approximations for the solutions of the Hermite differential equations

In the following link https://www.geogebra.org/m/fzyq89eg , you can see or download the applets designed to the solution and its approach to some Hermite differential equations.

Example 1:

Let

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 6y = 0, y(0) = 0, y(a) = 1, x \in [0, a]$$

Whose blue algebric solution in Figure 1 is $f(x) = \frac{3x-2x^3}{3a-2a^3}$, a varies between 0.1 and 1.

If $h = a/3$, then

$$\begin{aligned} x_0 &= 0 && \rightarrow \quad y_0 = 0 \\ x_1 &= a/3 && \rightarrow \quad y_1 = f(a/3) = \tfrac{a-\frac{2a^3}{27}}{3a-2a^3} \\ x_2 &= 2a/3 && \rightarrow \quad y_2 = f(2a/3) = \tfrac{2a-\frac{16a^3}{27}}{3a-2a^3} \\ x_3 &= a && \rightarrow \quad y_3 = 1 \end{aligned}$$

The splin polynomials are in red Figure 1 as follows

$$S_0(x) = \left(\frac{2a - \frac{2a^3}{27}}{3a - 2a^3}\right)\frac{3}{a}x, x \in [0, a/3].$$

$$S_1(x) = \frac{a - \frac{2a^3}{27}}{3a - 2a^3} + \left(\frac{a - \frac{14a^3}{27}}{3a - 2a^3}\right)\left(\frac{3}{a}x - 1\right), x \in [a/3, 2a/3].$$

$$S_2(x) = \frac{2a - \frac{16a^3}{27}}{3a - 2a^3} + \left(1 - \frac{2a + \frac{16a^3}{27}}{3a - 2a^3}\right)\left(\frac{3}{a}x - 2\right), x \in [2a/3, a].$$

If $a = 1$, in $[0, 1/3]$ the absolute mistake is 0.02851; in $[1/3, 2/3]$ the absolute mistake is 0.08359; in $[2/3, 1]$ the absolute mistake is 0.13904

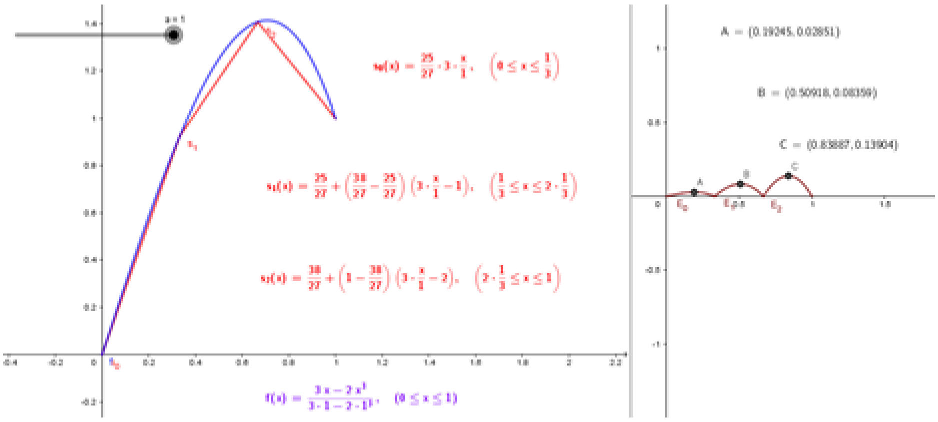

Figure 1.

Example 2
Let

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 10y = 0, y(0) = 0, y(a) = 1, x \in [0, a]$$

whose blue algebric solution in figure 2 is $f(x) = \frac{4x^5-20x^3+15x}{4a^5-20a^3+15a}$, with a varies between 0.1 and 5.
If $h = \frac{a}{3}$, then

$$\begin{aligned} x_0 &= 0 && \rightarrow \quad y_0 = 0 \\ x_1 &= a/3 && \rightarrow \quad y_1 = f(a/3) = \tfrac{4(a/3)^5-20(a/3)^3+15(a/3)}{4a^5-20a^3+15a} \\ x_2 &= 2a/3 && \rightarrow \quad y_2 = f(2a/3) = \tfrac{4(2a/3)^5-20(2a/3)^3+15(2a/3)}{4a^5-20a^3+15a} \\ x_1 &= a && \rightarrow \quad y_3 = 1 \end{aligned}$$

The splin polynomials are in red in figure 2, are as follows

$$
\begin{aligned}
S_0(x) &= \left(\frac{4(a/3)^5-20(a/3)^3+15(a/3)}{4a^5-20a^3+15a}\right)\frac{3}{a}x, x\in[0,a/3]\\
S_1(x) &= \left(\frac{4(a/3)^5-20(a/3)^3+15(a/3)}{4a^5-20a^3+15a}\right)\\
&+\left(\frac{4(2a/3)^5-20(2a/3)^3+15(2a/3)}{4a^5-20a^3+15a}-\frac{4(a/3)^5-20(a/3)^3+15(a/3)}{4a^5-20a^3+15a}\right)\left(\frac{3}{a}x-1\right), x\in[a/3,2a/3]\\
S_2 &= \frac{4(2a/3)^5-20(2a/3)^3+15(2a/3)}{4a^5-20a^3+15a}+\left(1-\frac{4(2a/3)^5-20(2a/3)^3+15(2a/3)}{4a^5-20a^3+15a}\right)\left(\frac{3}{a}x-2\right), x\in[2a/3,a]
\end{aligned}
$$

If $a = 2.5$, then $[0, a/3]$ the absolute mistake is 0.03146; $[a/3, 2a/3]$ the absolute mistake is 0.37258; $[2a/3, 1]$ the absolute mistake is 0.01981

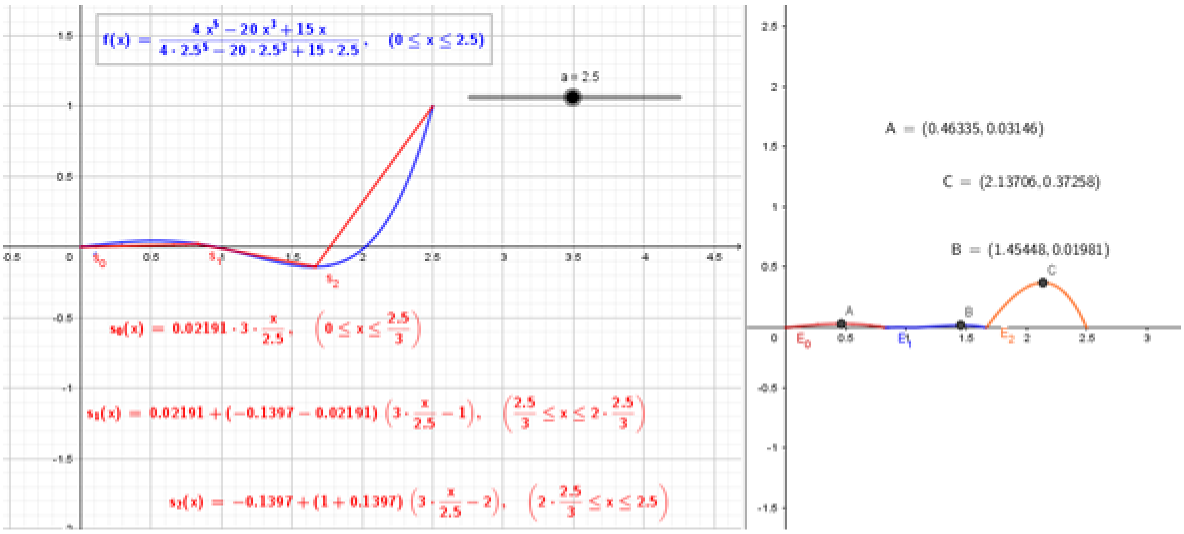

Figure 2.

Example 3
Let

$$\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+8y=0, y(0)=0, y(a)=1, x\in[0,1]$$

whose blue algebric solution in figure 2 is $f(x) = a\left(\frac{4x^4}{3}-4x^2+1\right)$, with a varies between -5 and 5.

If $h = \frac{1}{3}$, then

$$
\begin{aligned}
x_0 &= 0 & \rightarrow\quad & y_0 = a\\
x_1 &= 1/3 & \rightarrow\quad & y_1 = f(1/3) = \tfrac{139a}{243}\\
x_2 &= 2a/3 & \rightarrow\quad & y_2 = f(2/3) = \tfrac{-125a}{243}\\
x_3 &= 1 & \rightarrow\quad & y_3 = \tfrac{-5a}{3}
\end{aligned}
$$

The splin polynomials are in red in figure 3, are as follows

$$
\begin{aligned}
S_0(x) &= a + \left(\frac{139a}{243} - a\right) 3x, x \in [0, 1/3] \\
S_1(x) &= \frac{139a}{243} + \left(\frac{-125a}{243} - \frac{139}{243}\right) (3x - 1/3), x \in [1/3, 2/3] \\
S_2 &= \frac{-125a}{243} + \left(\frac{-5a}{3} - \frac{-125a}{243}\right) (3x - 2/3), x \in [2/3, 1]
\end{aligned}
$$

If $a = -0.2$, then $[0, 1/3]$ the absolute mistake is 0.02079; $[1/3, 2/3]$ the absolute mistake is 0.01104; $[2a/3, 1]$ the absolute mistake is 0.00929

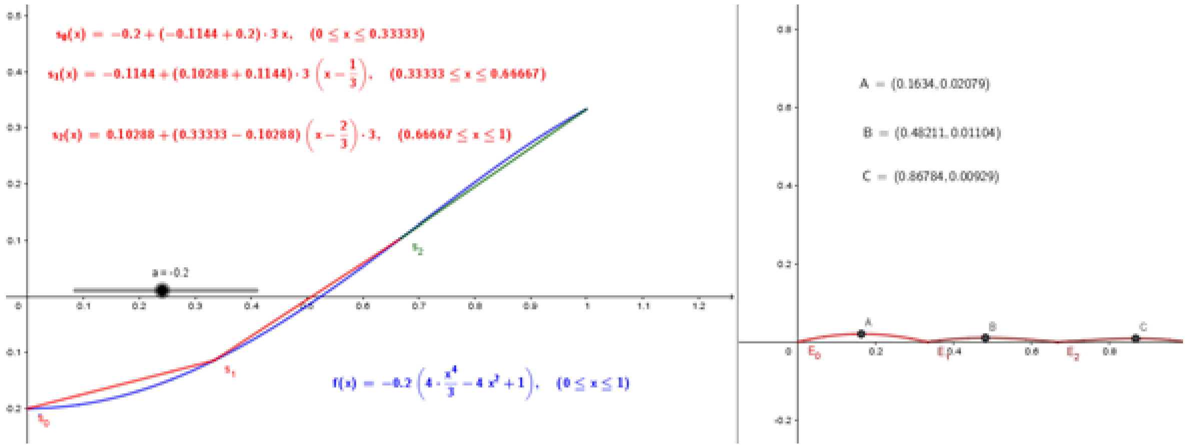

Figure 3.

3. Conclusion

We can say that the use of GeoGebra software allows us to understand the grade 1 spline method, interactively and pleasantly to be able to visualize the good and not so good approaches to the solutions of Hermite differential equations. At the same time we that we see not only the solution and spline approximation polynomials that are generated, but also the absolute error, which occurs in each corresponding sub-network. We look forward to this paper as being able to continue advancing in various lines of numerical analysis and mathematical physics supported by GeoGebra [2], [3].

4. References

[1] López, Armando (**2008**). *Propuesta para la enseñanza del concepto de derivada, un acercamineto visual con GeoGebra.* Patricia (Ed.), Acta Latinoamericada de Matemática Educativa pp 1166-1175, México DF. Comité Latinoamericano de Matemática Educativa A.C.

[2] Olivares, Jorge and Valero, Elvis (**2018**). *Animation and intereactive creations in linear diferential equations of first order: the case GeoGebra.* Phy. Conf. Ser. 1141:012126.

[3] Caligaris M, Schivo and Romiti, R. (**2015**).*Calculus & GeoGebra an interesting partnership.* Procedia Soc. Behav. Sciencies 174, 1183-1188.