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To cite this article: Jorge Olivares Funes and Elvis Valero Kari 2019 *J. Phys.: Conf. Ser.* **1391** 012057

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Resolving non-homogeneous linear differential equations using the undetermined method coefficients and variation of parameters by means of GeoGebra

Jorge Olivares Funes

Department of Mathematics, Universidad de Antofagasta, Antofagasta, Chile

E-mail: jorge.olivares@uantof.cl

Elvis Valero Kari

Department of Mathematics, Universidad de Tarapacá, Arica, Chile

E-mail: evalero@uta.cl

Abstract. In this paper, it is shown how non-homogeneous linear differential equations, especially those of the second order, are solved by means of GeoGebra applets. This is done by indeterminate coefficient methods and variation of parameters, in the course of differential equations for engineering students of the University of Antofagasta in Chile. The use of free software GeoGebra has been increasing in the teaching of mathematics, mainly in non-homogeneous linear differential equations, because it facilitates the teaching and learning process.

1. Introduction

The teaching and learning of the non-homogeneous linear differential equations of Second Order by means of the indeterminate coefficient methods and variation of parameters in the engineering courses of the University of Antofagasta has always been a very complicated subject for the students, this taking this taking advantage that the GeoGebra is a free use of software of mathematics for higher education, available in multiple platforms. Allows dynamic and interactive learning and visual differential equations see [1] Reminding that this mathematical visualization is the process of forming images and use them effectively for discovery and mathematical understanding, and consider of the visual as a prelude to the abstraction of concepts and thus allow the student to form several models of a learning situation see [6], [7] and [8] The visualization of differential equations and their graphical and algebraic solutions through GeoGebra applets makes it possible to create the sensation of self-evidence and immediacy, this is typical of the dynamic learning environment offered by GeoGebra. The use of sliders of the applets of this software allows the student to transform an equation with its general solution into infinite others through multiple representations in real time, In this way it facilitates the approximation to the behavior of differential equations and how they are going to be solved either by indetermined coefficients or variation of parameters, Through manipulation, exploration and



experimentation the student extracts his own conjectures, ideas and conclusions, achieving a more lasting and significant learning [1- 3].

Also as stated by the National Council of Teachers of Mathematics (NCTM), (2000) “When technological tools are available, students can focus their attention on processes of decision making, reflection, reasoning and problem solving.” [4,5]. The objective of this work is to show a with several animated applets, examples of non-homogeneous second order linear differential equations, emphasizing their solutions. These tools were designed as part of a Teaching Project for the course of Differential Equations of the Mathematics Department of the Faculty of Basic Sciences at the University of Antofagasta. It is possible to use or download these Applets and others on the website <https://edolinealysusmetodosprofejorge.blogspot.com> [9], [10].

2. The Applets Geogebra

The GeoGebra application is an interactive and free software that combines various branches of mathematics, and that allows diverse representations of mathematical objects . This facilitates the visualization of a non-homogeneous second-order linear differential equation taught in the course of differential equations, the following applets and examples are prepared

Example 1. Here it is considered the differential equation

$$\frac{d^2y}{dx^2} - (a + b)\frac{dy}{dx} + aby = x + 1$$

where a is real constant that moves between 0.5 to 3 and b is another constant between 3 and 5.

The constants c_1 and c_2 assume values between -5 and 5.

In Figure 1 it is shown the method of undetermined coefficients, and the particular solution is

$$y_p = \frac{x}{ab} + \frac{a + b}{(ab)^2}$$

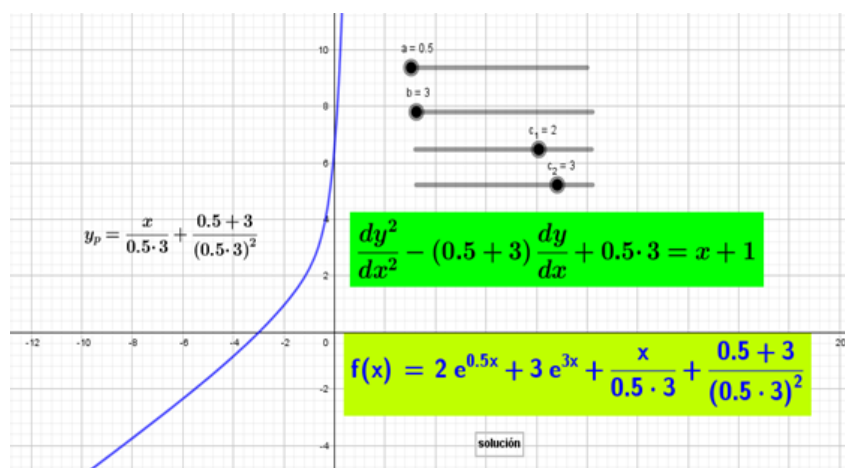


Figure 1. The graphical solution of $f(x)$ and algebraic

Example 2. *It is considered*

$$\frac{d^2}{dx^2} - (a + b)\frac{dy}{dx} + ady = e^x$$

where a is a real constant that moves between 1.1 to 5, and b is another constant between 1.1 and 15.

The constants c_1 and c_2 assume values between -5 and 5. In figure 2, the method of indeterminate coefficients is shown by the particular solution

$$y_p = \frac{e^x}{1 - (a + b) + ab}$$

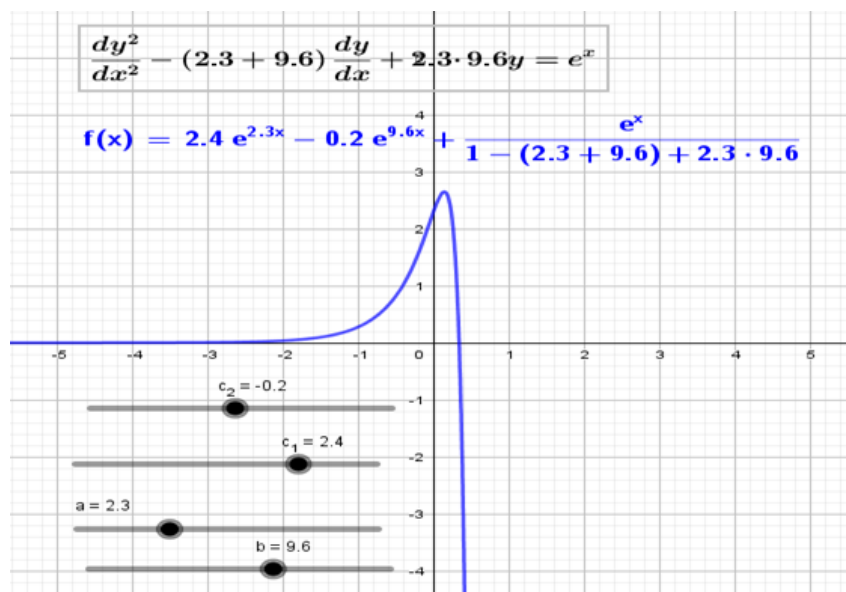


Figure 2. The graphical solution of and algebraic are in blue color.

Example 3. *It is considered*

$$\frac{d^2y}{dx^2} + w^2y = 1$$

where w is a real constant and with the slider moves between 0.2 and 6.2 The constants c_1 and c_2 assume values between -5 and 5. In Figure 3, the particular solution by variation of parameter is $\frac{1}{1w^2}$ is shown

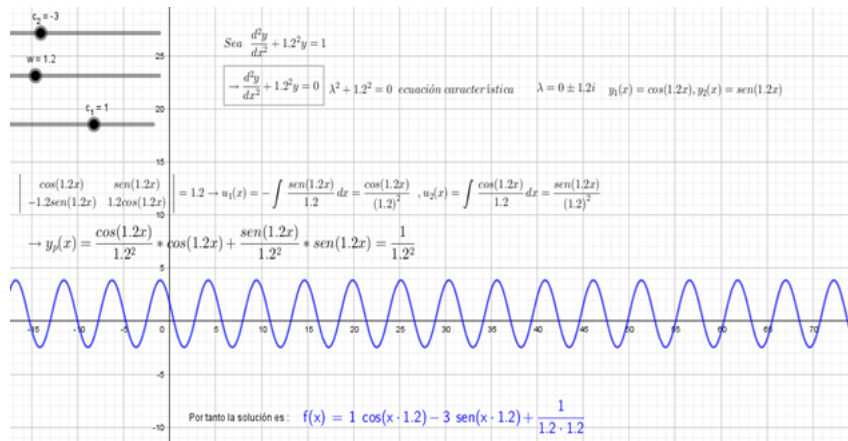


Figure 3. The graphical solution of f(x) and algebraic.

Example 4. Here the differential equation

$$\frac{d^2y}{dx^2} + w^2y = w$$

where w is a real constant real and with the slider that moves between 0.2 and 6.2.

The constants c1 and c2 assume values between -5 and 5. In Figure 4, the particular solution by variation of parameters $\frac{1}{w}$ is shown

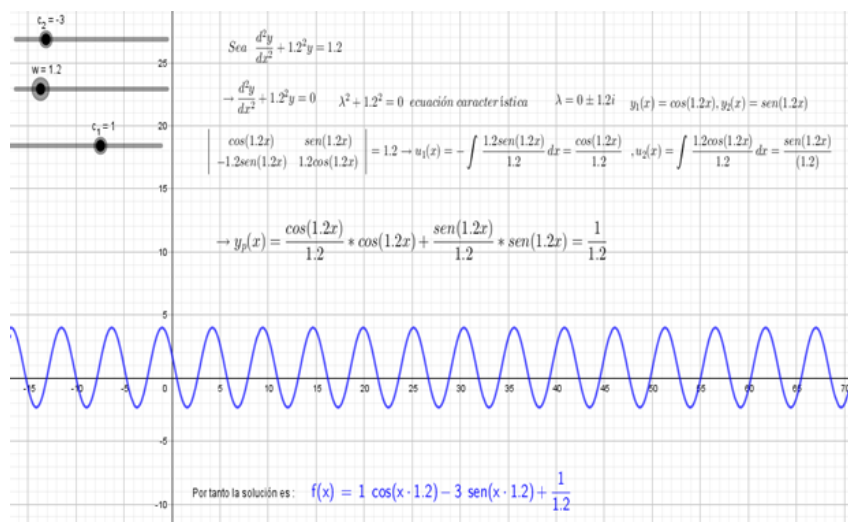


Figure 4. The graphical solution of f(x) and algebraic.

3. Conclusion

When reflecting on the teaching and learning strategies of linear second-order differential equations for indeterminate coefficients and variation of parameters, we must take into account the existence of GeoGebra as free software with dynamic capacities and interactive representation, Since it helps to improve and appreciate the content taught in a visual and animated way.

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