Sixth Order Linear Differential Equations Using GeoGebra Applets

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Abstract.
In Mathematics, the representations are understood as symbolic notations, graphics or verbal expressions, through which the most relevant concepts, procedures, characteristics and properties in this science are expressed. In this paper, in the context of Duval's Theory of Semiotic Representations, we use GeoGebra Applets as a teaching innovation tool that allows us to describe a form of conversion between algebraic and graphical semiotic representation registers of the solutions of sixth order linear differential equations, with the purpose of facilitating the process of its understanding and analysis.

INTRODUCTION

According to Duval [1], mathematics learning is a field of study appropriate to the analysis of important cognitive activities such as conceptualization, reasoning, problem-solving, and text comprehension.

In Mathematics, the representations are understood as symbolic notations, graphics or verbal expressions, through which the most relevant concepts, procedures, characteristics and properties in this science are expressed. In this context, Duval [2] indicates that these representations are grouped into different registers of representation according to their characteristics. In particular, the Theory of Semiotic Representation Registers developed by Duval explains the level of conceptualization based on the changes between the different registers of representation, requiring knowledge, treatment, and conversion of these registers, to be used in the different activities proposed. Thus, Duval indicates that the conceptual acquisition of a mathematical object is based on two of its strong characteristics: (i) the use of several registers of semiotic representation, (ii) the creation and development of new semiotic systems is a symbol of progress in knowledge.

For more information on Duval's Theory of Semiotic Representations see [3], [4], [5] and their respective references.

We will denote a linear differential equation of n-th order as

$$a_n(x)y^{(n)}(x)+a_{n-1}(x)y^{(n-1)}(x)+\ldots+a_2(x)y^{(2)}(x)+a_1(x)y^{(1)}(x)+a_0(x)y(x)=g(x) \tag{1}$$

where $a_n(x),\ldots,a_0(x),g(x)$ are continuous functions on an interval I and for all $x\in I$, $a_n(x)\neq 0$.
Let $y_1(x),y_2(x),\ldots,y_n(x)$ be a fundamental set of solutions of the homogeneous differential equation of order n that follows from (1) on the interval I. Then the general solution of the equation on the interval I is given by

$$y_G(x)=c_1y_1(x)+c_2y_2(x)+\ldots+c_ny_n(x) \tag{2}$$

where $c_1,\ldots,c_n$ are arbitrary constants. In our study, we will consider the particular case of $n=6$, that is, we will analyze the linear differential equation

$$a_n(x)y^{(6)}(x)+a_{n-1}(x)y^{(5)}(x)+\ldots+a_2(x)y^{(2)}(x)+a_1(x)y^{(1)}(x)+a_0(x)y(x)=g(x) \tag{3}$$

where its general solution on I is

$$y_G(x)=c_1y_1(x)+c_2y_2(x)+\ldots+c_6y_6(x). \tag{4}$$

For more information on differential equations, see [6].

GeoGebra is a computer program to support the teaching and learning of Mathematics for different stages of education, especially in geometry, algebra, and statistics. This software allows its users to visualize abstract geometric objects quickly, accurately, and efficiently. For more information about GeoGebra as a teaching tool see [3], [7], [8] and their respective references.

The present work is within the framework of the Núcleo de Investigación en Docencia Universitaria (NIDU 2-001-22) of the Universidad de Antofagasta (Chile), called "Applets de Geogebra para la docencia en Matemática". Considering Duval's Theory of Semiotic Representations, researchers from this nucleus belonging to the Department of Mathematics of the Universidad de Antofagasta (Chile) have decided to use GeoGebra Applets as a teaching innovation in learning algebraic and graphical representations of the solutions of sixth order linear differential equations.

REPRESENTATIONS OF SOLUTIONS OF SIXTH ORDER LINEAR DIFFERENTIAL EQUATIONS USING GEOGEBRA APPLETS

In this section, we will show examples of the use of GeoGebra Applets as an innovative teaching tool to visualize simultaneously the analytical and geometric representations of solutions of different sixth order linear differential equations. These Applets are free of charge and are available in https://www.geogebra.org/m/mtbacuyj.

In particular, in the following examples we will consider a parameter $a \in \mathbb{R} - \{0\}$. In the figures associated to each example, the differential equation shown in red, and the analytical solution and its corresponding graph in blue, where the sliders a and c_i with $i = 1, \ldots, 6$ vary in the interval $[-5, 5]$ in GeoGebra Applets.

Example 1 *Let*

$$y^{(6)}(x) - a^4 y^{(2)}(x) = x \tag{5}$$

where the general solution of the equation (5) is

$$y_G(x) = -\frac{a^2 c_2 e^{-ax} + a^2 c_4 e^{ax} + a^2 c_3 sin(ax) + a^2 c_1 cos(ax) + \frac{x^3}{3}}{2a^4} + c_6 x + c_5$$

where $a \in \mathbb{R} - \{0\}$ and the coefficients $c_i \in \mathbb{R}$, $i = 1, \ldots, 6$.

FIGURE 1. A solution of the differential equation (5) when $a = 0.8$

Example 2 *Let*

$$y^{(6)}(x) - 2ay^{(5)}(x) = 0 \tag{6}$$

where the general solution of the equation (6) is

$$y_G(x) = \frac{c_1 e^{2ax}}{a^5} + c_6 x^4 + c_5 x^3 + c_4 x^2 + c_3 x + c_2$$

where $a \in \mathbb{R} - \{0\}$ *and the coefficients* $c_i \in \mathbb{R}$, $i = 1, \ldots, 6$.

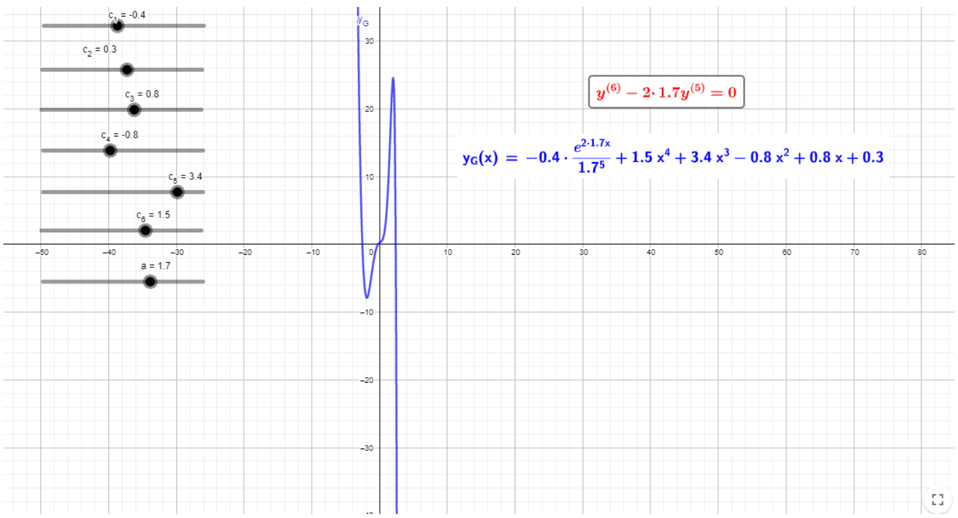

FIGURE 2. A solution of the differential equation (6) when $a = 1.7$

Example 3 *Let*

$$y^{(6)}(x) + a^4 y^{(4)}(x) = 0 \tag{7}$$

where the general solution of the equation (7) is

$$y_G(x) = \frac{c_2 sin(a^2 x)}{a^8} + \frac{c_1 cos(a^2 x)}{a^8} + c_6 x^3 + c_5 x^2 + c_4 x + c_3$$

where $a \in \mathbb{R} - \{0\}$ *and the coefficients* $c_i \in \mathbb{R}$, $i = 1, \ldots, 6$.

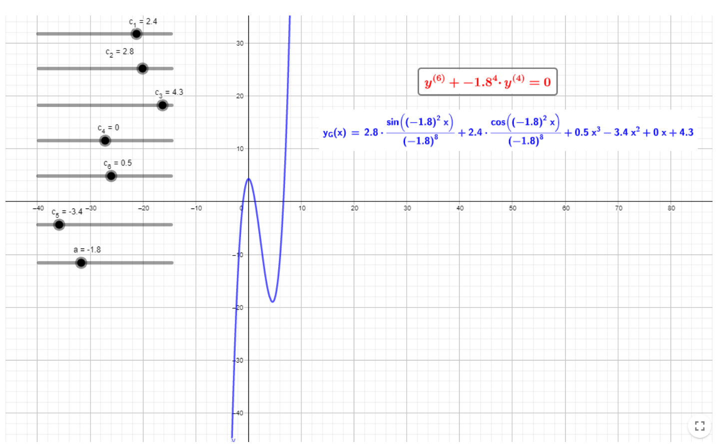

FIGURE 3. A solution of the differential equation (7) when $a = -1.8$

Example 4 *Let*

$$y^{(6)}(x) - 36y^{(4)}(x) = cos(ax) \tag{8}$$

where the general solution of the equation (8) is

$$y_G(x) = -\frac{cos(ax)}{a^4(a^2+36)} + c_6x^3 + c_5x^2 + c_4x + \frac{c_1e^{6x} + c_2e^{-6x}}{1296} + c_3$$

where $a \in \mathbb{R} - \{0\}$ *and the coefficients* $c_i \in \mathbb{R}$, $i = 1, \ldots, 6$.

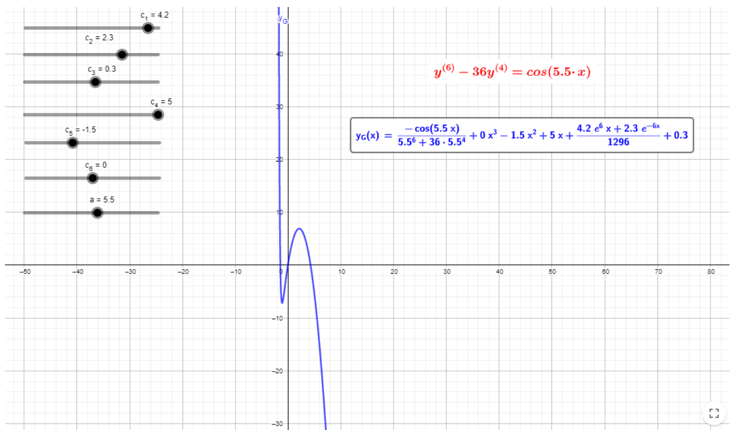

FIGURE 4. A solution of the differential equation (8) when $a = 5.5$

Example 5 *Let*

$$y^{(6)}(x) + a^2y^{(4)}(x) = x^2 \tag{9}$$

where the general solution of the equation (9) is

$$y_G(x) = \frac{c_2sin(ax)}{a^4} + \frac{c_2cos(ax)}{a^4} - \frac{x^4}{12a^4} + \frac{x^6}{360a^2} + c_6x^3 + c_5x^2 + c_4x + c_3$$

where $a \in \mathbb{R} - \{0\}$ *and the coefficients* $c_i \in \mathbb{R}$, $i = 1, \ldots, 6$.

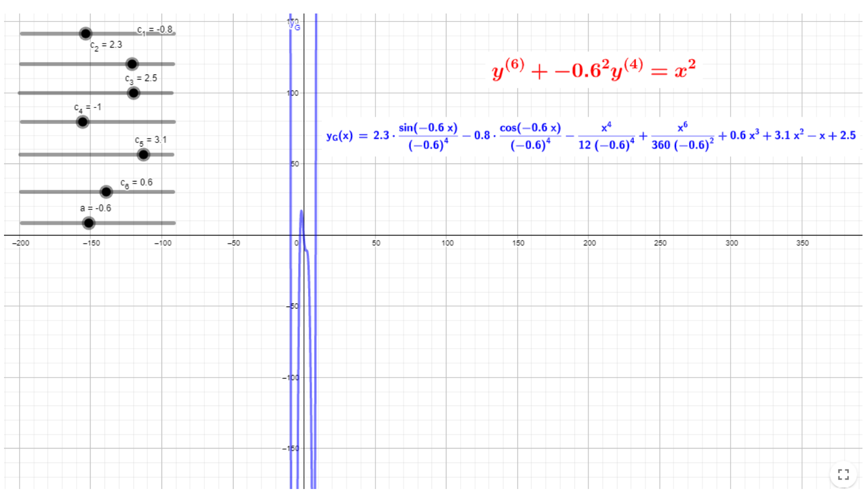

FIGURE 5. A solution of the differential equation (9) when $a = -0.6$

CONCLUSION

In mathematics, one way to analyze a phenomenon modeled by a differential equation is to study the general solution of the corresponding equation.

For a better understanding and analysis of the solutions, mathematical computer programs are playing a fundamental role. Thus, the use of GeoGebra Applets as a teaching innovation tool helps to strengthen the learning of students in the course of differential equations in the study, analysis, and understanding of the solutions of linear differential equations of sixth order, considering the registers of algebraic and graphical semiotic representations of these solutions. It should be noted that this tool is freely accessible and easy to understand, contributing directly to the academic training of students, who will be able to apply this knowledge to various problems in their areas of expertise.

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