

Special Cases in the Method of Undetermined Coefficients With GeoGebra in Non-Homogeneous Linear Differential Equations of Third Order

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Abstract. Over the years of teaching differential equations in engineering courses at the University of Antofagasta in Chile, the topic of the method of undetermined coefficients to find a particular solution in third order linear differential equations has always been one of the topics of learning complexity, this has been presented in the determination of the general solution to differential equations of third order. In this paper, the GeoGebra applets have been built to strengthen and support the dynamic learning of this method, and this has given excellent results.

Keywords: Differential Equations; Engineering; Education; GeoGebra; Tics

INTRODUCTION

Currently, the use of Information and Communication Technologies (ICT) is closely linked to the educational field through the various software existing for learning.

In previous work (1, 2), something very crucial is pointed out, and this is that we existing connection between Information and Communication Technologies (ICT), as a motivation in learning Mathematics. Together with this fact as mentioned in ref (3), that ICT promotes motivation in students, and this leads to improvements in learning and use of their free time. Within the Tics, in the context of mathematics learning, we highlight the GeoGebra software.

In ref (4), the construction of GeoGebra applets were made to visualize the particular solutions of linear second order equations. Now we will present the construction, but of linear third order differential equations with selected examples of the method of indeterminate coefficients. This has been used during the second semester 2021, and also in the first semester of 2022 on the subject of differential equations for civil engineering of the Universidad de Antofagasta.

INDETERMINATE COEFFICIENTS IN LINEAR DIFFERENTIAL EQUATIONS OF THIRD ORDER USING GEOGEBRA APPLETS

In this link <https://www.geogebra.org/m/rdjbewct>, the GeoGebra applets were built to strengthen and support the learning of the method of indeterminate coefficients in linear differential equations of the third order, and they are available for free viewing and downloading.

Example 1

Let us consider

$$y^{(3)}(x) - ay''(x) + 4y'(x) - 4ay(x) = x$$

Whose particular solution is

$$y_p = \frac{-x}{4a} - \frac{1}{4a^2}$$

The indeterminate coefficients are in red in Figure 1.

The slider "a" varies from 0.1 to 50.

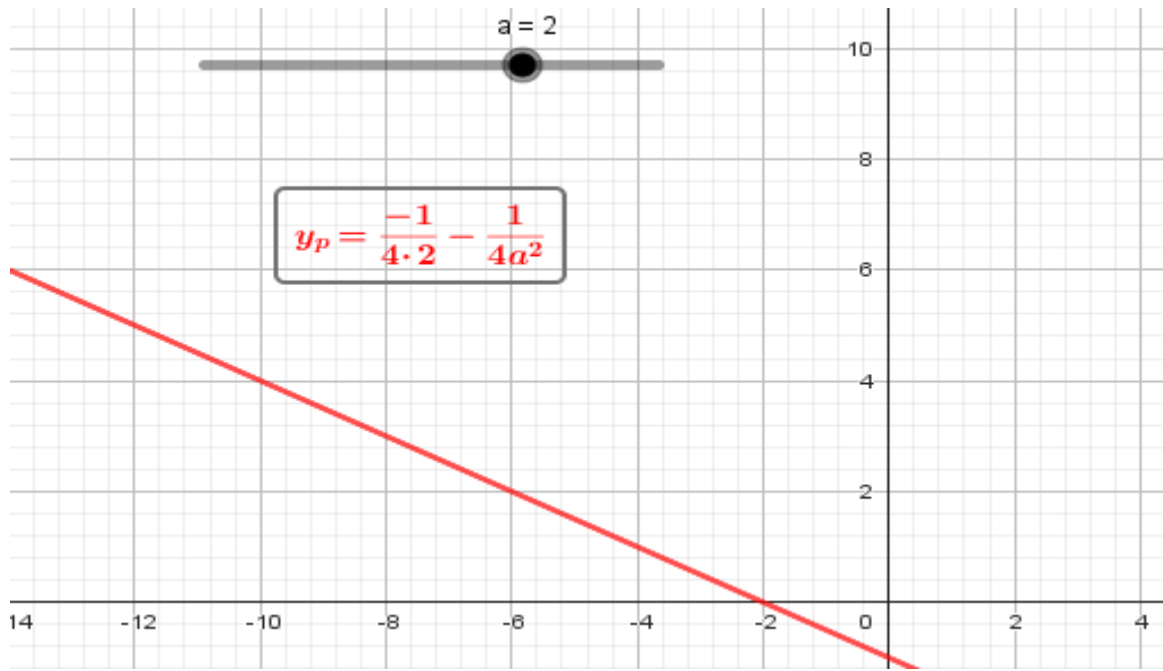


FIGURE 1. Particular solution in red

Example 2

Let us consider

$$y'''(x) - 2ay''(x) + a^2y'(x) = x + a$$

whose solution is

$$y_p = \frac{x^2}{2a^2} + \frac{a^2 + 2}{a^3}$$

where the indeterminate coefficients are in red in Figure 2.

The slider "a" varies from 0.1 to 50.

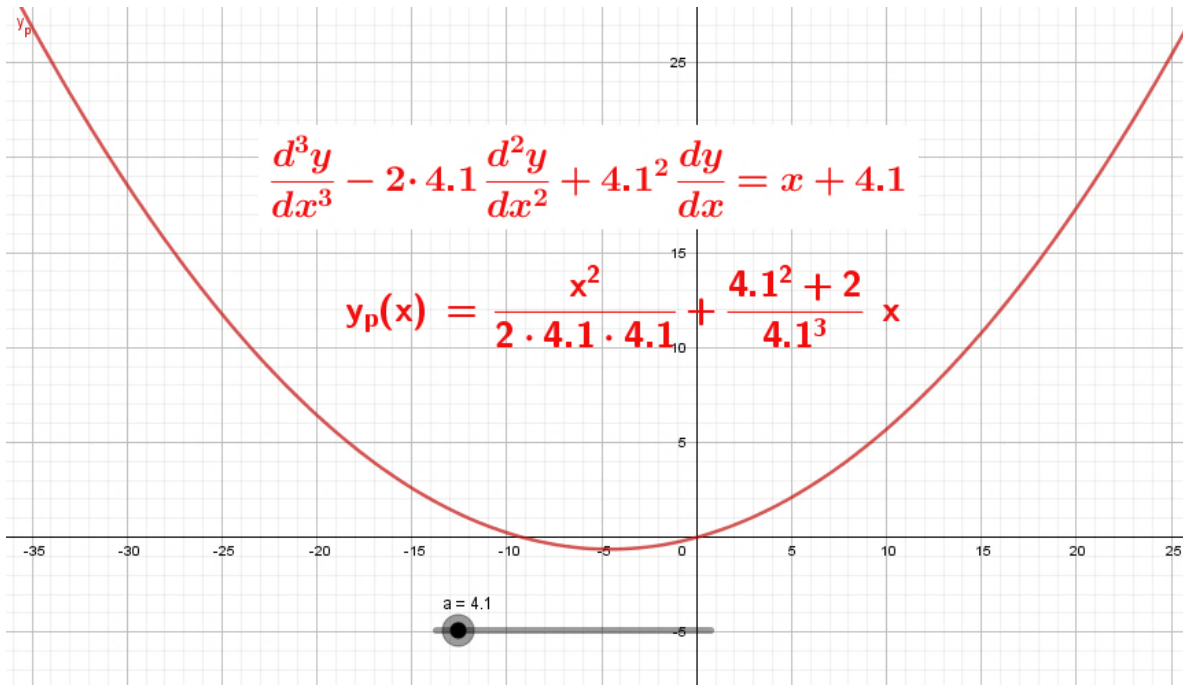


FIGURE 2. Particular solution in red

Example 3

Let us consider now

$$y^{(3)}(x) - 3ay''(x) + 3a^2y'(x) - (a^3 + ab^2)y(x) = ax$$

whose particular solution is

$$y_p = \frac{-x}{a^2 + b^2} - \frac{3a}{(a^2 + b^2)^2}$$

and the indeterminate coefficients are in red in Figure 3.

The sliders "a" and "b" will be in the range from 0.1 to 50.

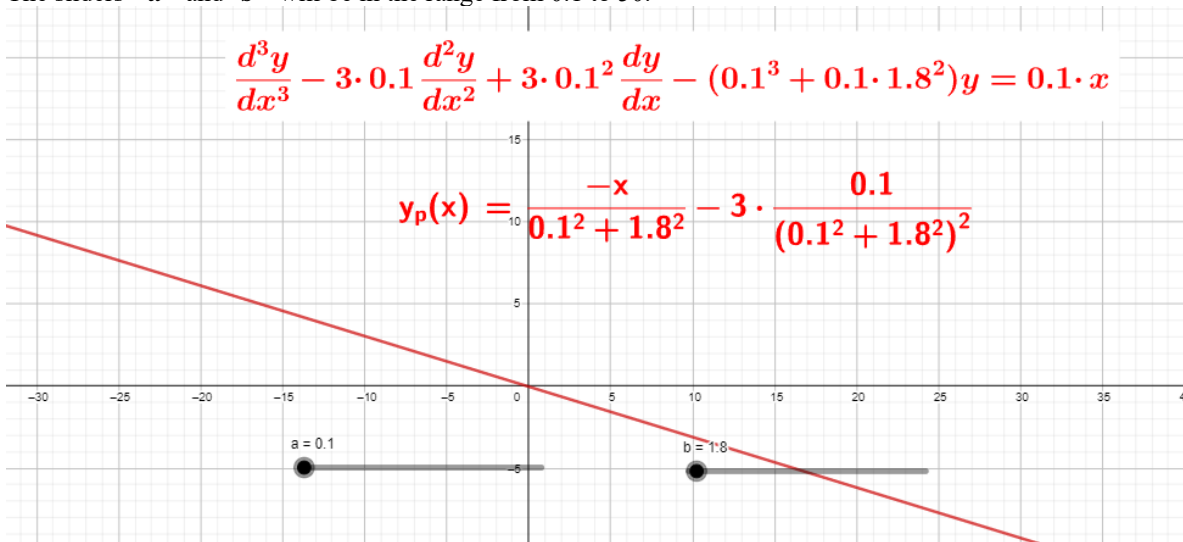


FIGURE 3. Particular solution in red

Example 4

The differential equation will now be

$$y^{(3)}(x) + a^2 y'(x) = \cos(x)$$

whose particular solution is

$$y_p = \frac{\sin(x)}{a^2 - 1}$$

and the indeterminate coefficients are in red in Figure 4.

The slider "a" varies between -50 to 0.9.

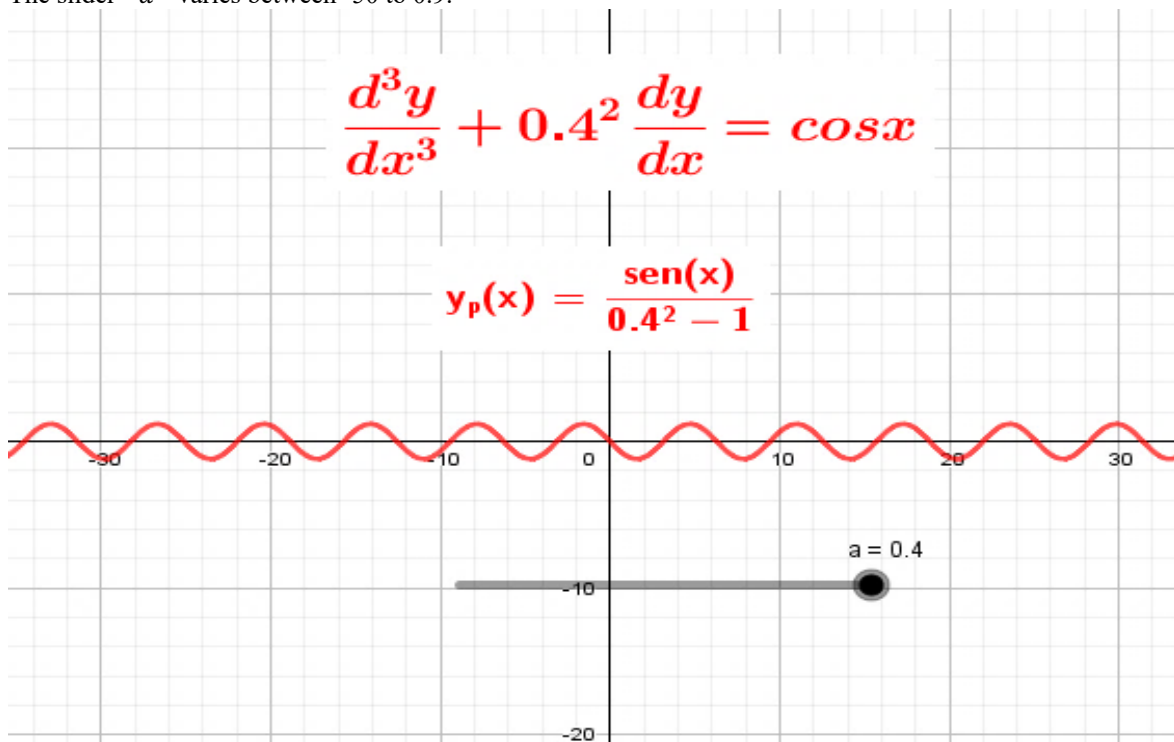


FIGURE 4. Particular solution in red

CONCLUSION

The use of GeoGebra software in the process of teaching and learning has been demonstrated and proven to have great effectiveness and support in the student and academic community (5, 6).

This software has reached a level of support in the development of learning in the various mathematics subjects and other areas much more than before and this is due to the pandemic.

We should also note that the motivation to achieve a better learning of the method of determined coefficients on the part of the students has improved notably and we hope to continue working with this software in different contents of differential equations or other subjects in the area of mathematics.

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