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Visualizing some numerical solutions of linear second order differential equations with the Euler method and Lagrange interpolation via GeoGebra

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Abstract. In this paper we will show the algebraic and graphic expressions, that were obtained through the Euler method and Lagrange interpolation by means of GeoGebra software for some linear second order differential equations. This teaching material was designed for the course of differential equations, and as a complement of support for the numerical calculation course for the engineering careers of the Universidad de Antofagasta.

Keywords: GeoGebra, Differential equations, Lagrange, Euler

1. Introduction

Formerly, the subjects of differential equations of the University of Antofagasta was usually in a very expository way, and the students were not very communicative with each other for the development of certain contents, as mentioned for example by some authors on this subject such as Enrique Vílchez Quesada, Eric Padilla Mora [1] as well as Meza and Hernández [2] who point out, “The processes that ordinarily develop in the teaching of Mathematics, are characterized by master classes, sequential presentation of content, additional practices, individualized work as a general norm and communication between the poor students ” (p.32)

With the passage of time, various educational software such as GeoGebra appeared that helped in the teaching and learning processes of mathematics, as Pereira [4] quoted in [3] who tells us that this software allows the student to “Explore an object while simultaneously viewing algebraic and geometric representations, drawing figures, manipulating them, exploring them, and this allows students to develop skills using their own strategies.”

It is for this reason that various GeoGebra applets were developed for the development of the classes and to motivate the students in their personal study schedule to see the numerical solutions obtained from some linear second-order differential equations using the method of Euler and Lagrange interpolation. This material was prepared for the differential equations course and also additionally for the numerical calculus course at the University of Antofagasta. This work is a continuation of Animations and interactive creations in first order linear differential equations:



the case of GeoGebra [5] Numerical solutions displayed by GeoGebra applets through Euler and Lagrange.

2. Numerical solutions displayed by GeoGebra applets through Euler and Lagrange

Next we will show some GeoGebra applets, where these numerical solutions are displayed along with the absolute error that occurs. All these applets are available in <https://www.geogebra.org/m/cq9mfyqg>

Example 1

Let us be

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} = ax^2, y(1) = a, \frac{dy}{dx}(1) = -a \tag{1}$$

whose cotton solution

$$y = \frac{a}{8}(2x^2 + 6 \sin(2 - 2x) + 7 \cos((2 - 2x) - 1))$$

and its graph is given in blue in figure 1, where a moves between 0.5 and 2.

By Euler's method with a variation $h = 0.1$, points A, B, C, D are obtained, then an approximate solution in red is formed by the Lagrange polynomial of order three with these points in the interval $[1, 1.3]$.

In red, the approximate solution algebraically and graphically.

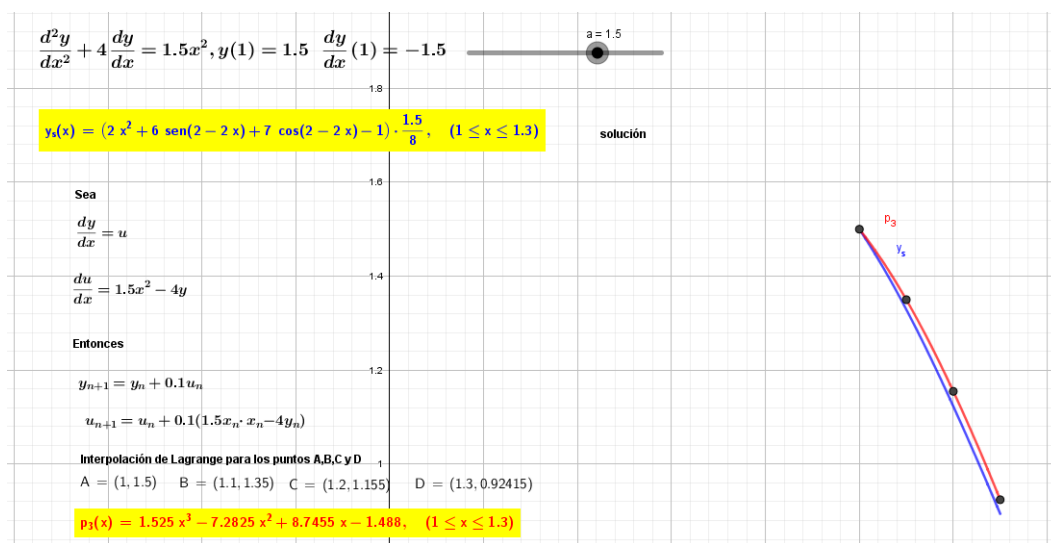


Figure 1.

Example 2

Let the equation

$$\frac{d^2y}{dx^2} - \left(\frac{x^2}{4} - \frac{1}{2}\right)y = 0, y(0) = a, \frac{dy}{dx}(0) = 0, \tag{2}$$

whose cotton solution $y = ae^{-\frac{x^2}{4}}$ and its graph is given in blue in figure 2, where a moves between 0.5 and 1.

By Euler’s method with a variation $h = 0.1$, points A, B, C, D are obtained. Then an approximate solution in red is formed by the Lagrange polynomial of order three with these points in the interval $[0, 0.3]$. In red, the approximate solution algebraically and graphically

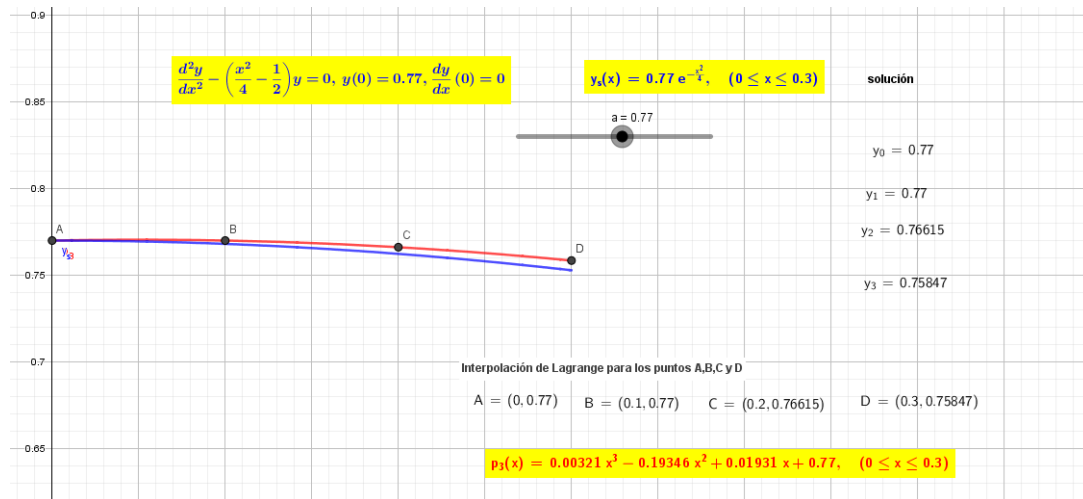


Figure 2.

Example 3

Let the equation

$$(1 - x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 56y = 0, y(0) = 0, \frac{dy}{dx}(0) = a, \tag{3}$$

whose cotton solution is $y = ay = a(-429 \frac{x^7}{35} + 99 \frac{x^5}{5} - 9x^3 + x)$ and its graph is given in blue in figure 3, where a moves between 0.1 and 0.5.

By Euler’s method with a variation $h = 0.1$, points A, B, C, D are obtained, then an approximate solution in red is formed by the Lagrange polynomial of order three with these points in the interval $[0, 0.3]$.

In red, the approximate solution algebraically and graphically.

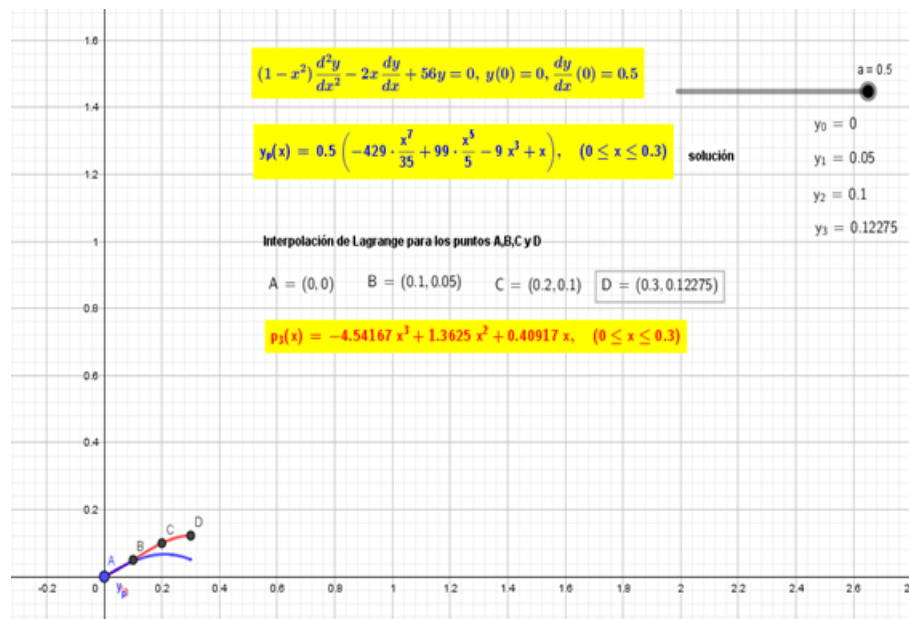


Figure 3.

3. Conclusion

GeoGebra applets as teaching material, allow us to see in an educational and motivational way the animations of the Lagrange and Euler approximations in the differential equations of linear second order, especially those of example 2 and 3 which are cylindrical parabolic and Legendre equations respectively. These equations have huge applications in Physics and Engineering today.

Acknowledgments

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