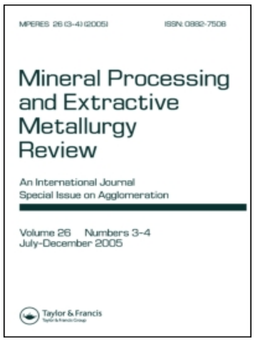

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A Methodology For Design And Operation Of Heap Leaching Systems

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ABSTRACT
Leaching is a hydrometallurgical activity widely used in mineral processing, both for metallic and non-metallic ores, and in soil remediation. The dissolution of valuable species by heap leaching is strongly dependent on the design and operating variables, so the study of the influence of these variables on recovery and their optimization for the best performance are attractive tasks for the development of the mining industry. In this work, a methodology is developed that enables the planning and design of leaching systems. This methodology uses a proposed superstructure and a mathematical model to analyze the system behavior and determine the optimal design and operating conditions. The model was generated with a Mixed Integer Nonlinear Programming (MINLP) approach and solved by different solvers under GAMS® software (General Algebraic Modelling System). The Spatial Branch-and-Bound (SBB) solver obtained the global optimum in the shortest times. Based on a case of study for copper leaching, it is demonstrated that the procedure allows achieving optimal design and operational conditions.

KEYWORDS
Copper; design; heap leaching; optimization; planning

1. Introduction

Heap leaching is a hydrometallurgical process that, due to its economic and environmental advantages, is considered as a standard treatment for the extraction of low-grade ores by the mining industry (Cathles and Apps, 1975; Gálvez et al., 2012; Ghorbani et al., 2016; Mousavi et al., 2006). Also, heap leaching has been recently studied as a technology to pollutant removal in the soil remediation (Hu et al., 2014). Heap leaching involves the dissolution of a soluble compound from a stacked ore or soil by using an external leachant, which extracts the valuable material from the solid matrix. In the mining industry, cyanide solutions are used for the processing of some precious metals as gold and silver, sulphuric acid solutions for copper and water for non-metallic ores as caliche (Trujillo et al., 2014; Valencia et al., 2008). Although heap leaching has been practiced for centuries, it is in the second half of the twentieth century where it begins to have a significant relevance in industrial scale, especially for the extraction of metals such as copper, gold, silver, zinc and caliche (Bartlett, 1997; Habashi, 2005; Ordóñez et al., 2014).

Because of its importance, heap leaching has become in the subject of multiple investigations that seek to predict the process performance and improve it under different approaches. Some mathematical models have been formulated to determine the effect of input variables of heap leaching through simulations, which allow to understand the phenomena in the process and predict results. These models can demonstrate that the choice of the input parameters may have a significant effect on the leaching performance (Bartlett, 1992; Mellado and Cisternas, 2008; Roman et al., 1974; Teles et al., 2013). Generally, the conducted studies analyze the process optimization only from a technical point of view, either through phenomenological approaches (Bennett et al., 2012; Dixon and Hendrix, 1993a, 1993b, McBride et al., 2012a, 2012b) that describe the system using complex equations, or through analytical approaches that start from simple constitutive equations (Bouffard and Dixon, 2009; Mellado et al., 2012, 2011a, 2011b, 2009). However, few models have performed a full system optimization (García et al., 2010; McBride et al., 2014), by analyzing the leaching from a comprehensive perspective, i.e. taking into account the process design, process planning, the operational variables, and their effect on the economic benefits.

Mathematical programming can provide information that allows planning the processing of minerals and facilitates the evaluation and the decision making, which is an essential tool for monitoring and process design (Coderre and Dixon, 1999). Regarding optimization and heap leaching operation, the technique is carried out until maximum recovery is obtained under the best economic conditions (Padilla et al., 2008). Many studies have proved the effect of other operations such as crushing, agglomeration and ventilation on the recovery, also the influence of flow multistage through porous media (Bartlett, 1997; Cariaga et al., 2005; Cross et al., 2006; de Andrade Lima, 2006; Mousavi et al., 2006; Sheikhzadeh et al., 2005). The balance between the recovery and the plant capacity has been analyzed from an economic point of view, identifying that there is an optimum in terms of design and planning (Padilla et al., 2008).

The problem of leaching system, defined by a set of heaps was studied by Trujillo et al. (2014), who developed a methodology through a superstructure that consists in a

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combination of all the possible streams among the unit processes. The method was applied to the extraction of copper in different heap leaching systems, giving results that allowed the analysis and design of these systems. The results showed that the system converged to an optimal solution using a Mixed Integer Nonlinear Programming (MINLP) model. In that case, the leaching time was considered as an operational variable and the heap height as a design variable. Moreover, the optimum design considered the maximum feasible height and leaching times that make a balance between efficiency (recovery) and capacity (number of leaching cycles in the evaluated horizon). However, the superficial velocity of leaching, or the irrigation rate, was not included in the analysis. This is an important drawback because the irrigation rate plays a major role in heap leaching. In fact, there is an optimal irrigation profile as was demonstrated by Mellado et al. (2011b).

In the process design, one of the critical aspects is the handling of nonlinear parameters, such as: pressure, temperature, concentration, conversion and split fraction. In this context, the use of linear models (MILP) is not always appropriate, since they have disadvantages associated to the need to discretize conditions, which leads to increase the number of binary terms. In this way, the application of nonlinear algorithms (MINLP) has grown in the field of process engineering, because on one hand, overcomes the problems of discretization and the other, extends the scope of the resolution through a continuous treatment of data (Grossmann, 1985; Kocis and Grossman, 1987). Although MINLP models are more complex, they are more suitable for problems similar to those posed in this study. The MINLP model presented by Trujillo et al. (2014) experienced convergence problems, and some scenarios did not converge at all.

The aim of this work is to develop a methodology that enables the planning and design of leaching systems, including irrigation rates, through a proposed superstructure and a mathematical model to analyze the system behavior and determine the optimal design and operating conditions. It also seeks to compare different solvers, both global and local, to identify those who are best suited to solve this model.

2. Proposed methodology

In this work, a set of alternatives for heap leaching systems is proposed by a superstructure, which is represented by a mathematical model. This model provides an approximation of the system behavior and the optimal solution from an economic point of view, determining the design (structure) and operation (leaching time and irrigation rates) conditions. Figure 1 shows the proposed superstructure, which represents the set of alternative flows that are distributed through the heaps. A heap leaching unit *jp* is represented by the structure in Figure 1 that is based on the work of Trujillo et al. (2014). L represents the mass flow and the subscripts i and j denote processing units. The general set of units and symbols used along the text are defined in the nomenclature section. A mixer at the entrance of the processing units and a divider at the output are considered. Heaps are arranged in series and flows between them and to a solvent extraction unit are allowed, but a stream output cannot enter to the same unit again. The solvent extraction unit receives the solution that comes from each heap its output feeds the heaps.

For one, two, and three heaps the number of alternatives of stream structures are 1, 27, and 2,401, respectively. The methodology finds the configuration that would lead to the optimal solution and the maximum value of the target function. The mathematical model designed in this study is proposed from a framework that incorporates the design and operational parameters with intrinsic economic factors of the process (Equation(1)). Here a brief description of the model is given. The complete mathematical model is given in the Appendix A. In general, the model corresponds to MINLP model of the form,

$$\begin{array}{c} Max\, f(x,d,p,y) \\ s.a.g(x,d,p,y) \leq 0 \end{array} \tag{1}$$

Where x represents operational variables such as concentration, mass flow rate, volumetric flow rate, and irrigation rate. The symbol d represents design variables such as heap height, p represents planning variables such as number of cycles and cycle time. All x, d, p variables are continuous variables. On the other hand, y represents binary variables that are used to represent decisions for the selection of cycle time and selection of disjunction in the linearization of the heap recovery model.

Models are needed to represent the copper recovery, $R_{jp,k}$ and acid consumption, $C^{+}_{H_{jp}}$ in heap *jp*, Equation (2) and Equation (3), respectively.

$$R_{jp,k} = f(z, us, r, t) \tag{2}$$

$$C^{+}_{H_{jp}} = f(z, us, r, t) \tag{3}$$

Where z is the heap height, us is the superficial velocity of irrigation, r is the particle radius, and t is the leaching time. First, the analytical model proposed by Mellado et al. (2011a, 2009) is used that includes the kinetics at particle level and at heap level. However, the Mellado model includes two exponential expressions that introduce nonlinear equations to the model (see Equations (A32) to (A37) of Appendix A). These

Figure 1. Generic superstructure of streams considered in one heap system. $L^{in}_{jp,k}$ and $L^{out}_{jp,k}$ represent the input and output mass of the k species in the heap *jp*. $L_{i,j,k}$ and $L_{j,i,k}$ correspond to the flows before and after the virtual mixing and splitting units, respectively.

nonlinear equations can produce difficulties in the optimization solution of the problem. For that reason a second model was included that is a linearization of the Mellado model (see Equations (A38) to (A47)). This linearization is done through disjunctive functions utilizing interval sections of time to approximate the predicted recoveries obtained from the Mellado model.

The mathematical model also includes mass and volumetric balance in heaps, mixers, dividers, and solvent extraction unit (see equations A1 to A5 and A22 to A26). The majority of these equations are linear equations, but the relationship between mass and volumetric flow rates is a bilinear expression (e.g. $Lx = Q$, where L, x, Q are mass flow rate, concentration and volumetric flow rate, respectively). Bilinear expression is nonconvex nonlinear and therefore can produce multiple local optimal solutions.

In a time horizon (H) several cycles of leaching (N) can be achieved, for example if a heap is leached by 90 days, then four cycles of leaching are performed in a time horizon of 360 days and the cycle time (t) is 90 days. This is $Nt = H$. However, if the leaching is performed in two heaps operated in series simultaneously, the cycle time will be the large time of leaching heap one and two. For the selection of cycle time the Big M method was applied to the disjunctions that represent the selection of the cycle time. These equations include binary variables, but are linear expressions. However, the equation $Nt = H$ is bilinear. All these equations can be considered as the planning model (see Equations (A6) to (A12)).

For improving the optimization performance of solvers and to reach logical solutions it is necessary to include lower and upper bounds for the variables. For example, the cycle time cannot be higher than the horizon time and copper recovery must be between 0 and 1 (see Equations (A13) to (A21)).

In this study, the optimization task involves the minimization of the operating costs and maximization of incomes to reach the maximum possible profits. The incomes are represented by the total production of the valuable species and its price. The costs are determined by the design, construction and operation of each heap. The profits (U) result from the subtraction of total incomes (I) minus total costs (C) (Equation (4)).

$$Maximize\ U = I - C \tag{4}$$

Incomes represent a set of intrinsic variables such as production (P_k), species price (P_{C_k}), number of cycles and post leaching costs ($C_{post-leac}$), expressed by:

$$I = NP_k\left(P_{C_k} - C_{post-leac}\right) \tag{5}$$

On the other hand, the costs include number of cycles, total cost for each cycle per area, the variable cost per ton of extracted mineral ($C_{v_{jp}}$), mineral density and design variables such as area and heap height (Equation(6)).

$$C_{jp} = C_{T\,jp} + C_{V\,jp}(\rho Az) + C_{pre-leac_{jp}}(\rho Az) \tag{6}$$

Where C_{jp} represents the construction and operation costs of heap. To determine the total cost of the process (C), the acid price (P_A) and the acid consumption of the heap jp (C_{A_T}) are included.

$$C = N\left(C_{A_T}P_A + \sum_{jp} C_{jp}\right) \tag{7}$$

3. Results and discussion

3.1. Performance analysis of solvers

Both Mellado and disjunctive mathematical models are nonconvex MINLP, and therefore it is difficult to solve for global optimum. For this reason, the performance of eight different MINLP solvers was studied, which are: AlphaECP, BARON, BONMIN, DICOPT, COUENNE, LINDOGlobal, OQNLP and SBB. A brief description about these solvers appears in Table 1. GAMS software was used for the optimization, and the solvers were compared regarding their efficiency and resolution time.

The Mellado and Disjunctive models were solved for systems comprised by one, two and three leaching heaps. In the case of the disjunctive model, four (D4) and eight (D8) disjunctions were carried out to evaluate the effectiveness to use different number of disjunctions. The effect of copper price on the profits was evaluated in this study.

The efficiency was determined by evaluating the capability of each solver to converge and to get to the optimum value. In the case of the disjunctive model, for four disjunctions, cycle time intervals of 40 days were obtained by the optimization program, while for eight disjunctions intervals of 20 days were reached. The parameters considered in the performance analysis of solvers are shown in Table 2.

For one heap system, the obtained solution by the different MINLP solvers is almost the same, except AlphaECP and OQNLP that showed non-convergence for both models (Table 3). In general, as the number of heaps increases, the convergence problems are larger. For two and three heap systems some solvers have no convergence, or local solutions were obtained. OQNLP worked well for some cases using the Mellado model but not for the disjunctive one. On another side, AlphaECP cannot solve the Mellado model, while LINDOGlobal obtained local optimal for some cases. It is important to note that some of the aspects that determine if a solver can reach an optimal solution are the algorithms and the nonlinearities processing techniques that it has, which can

Table 1. MINLP solvers used in this study.

Solver	Algorithm	Type of optimum obtained
AlphaECP	Extended-cutting-plane method based for pseudo-convex problems.	Global
BARON	Specialized branch-and-reduce deterministic algorithms.	Global
BONMIN	Branch-and-bound and outer-approximation based.	Local
DICOPT	MILP/NLP outer-approximation algorithm.	Local
COUENNE	Branch-and-bound and outer approximation based for pseudo-convex problems.	Global
LINDOGlobal	Branch-and-bound and outer approximation based algorithm.	Global
OQNLP	Multi-start heuristic algorithms.	Local
SBB	NLP based branch-and-bound algorithm.	Local

Table 2. Operational parameters used in the performance analysis of MINLP solvers.

Parameter	Value
Mineral density, ρ (t/m^3)	1.7
Ore grade, λ_k (%)	0.9
Time horizon, H (d)	360
Heap area, A (m^2)	200,000
Heap height, z (m)	9.5
Post leaching cost, $C_{post-leac}$ (kUS$/t Cu)	0.17
Copper price, P_{C_k} (kUS$/t Cu)	7.7
Acid price, P_A (kUS$/t H_2SO_4)	0.16
Variable cost, $C_{V_{jp}}$ (kUS$/t ore)	0.0029

be: Branch and Bound, Outer Approximation, Generalized Bender Decomposition and Extended Cutting Plane.

In this work, all the techniques are addressed by the employed solvers and the differences obtained in their performance depends on its formulation. According to the results, the solvers that achieved global optimum values were: BARON, BONMIN, COUENNE, and SBB. These solvers have in common that they are based on the Branch and Bound algorithm, which appears to be better for this type of problem. Taking it into account, these four solvers were considered for the subsequent study of the execution time.

The variables considered in the execution time analysis were: copper price, ore grade, variable cost, and acid price. These variables were analyzed in three levels, according to the values shown in Table 4.

The execution time was measured in a computer with an operative system of 64 bits, processor Intel® Core i5 3.10 GHz and RAM memory of 4 GB. The analysis was performed summing the resolution times of each one of the three levels for the four studied variables (copper and acid price, variable cost and ore grade) for both disjunctive (4 and 8 disjunctions) and Mellado models. The results are given in Table 5, which corresponds to the addition of the execution times of the 12 cases indicated in Table 4.

The execution times were consistent between the run of the 4- and 8- disjunctions for the disjunctive model. As expected, the number of heaps significantly increases the run times in a non-lineal trend. For one heap, the disjunctive model solved 68 equations, 63 variables and 17 binary variables, while for three heaps the model has 238 equations, 196 variables and 54 binary variables. On the other hand, in Mellado model for

Table 4. Variables and levels used in the execution time analysis and study of cases.

Parameter	Lower value	Medium value	Large value
Copper price, P_{C_k} (kUS$/t Cu)	5.5	7.7	9.9
Acid price, P_A (kUS$/t acid)	0.110	0.162	0.210
Variable cost, $C_{V_{jp}}$ (kUS$/t ore)	0.005	0.010	0.020
Ore grade, λ_k (%)	0.5	0.9	1.3

Table 5. Additive execution times for the 12 cases showed in Table 5, obtained for the Disjunctive and Mellado models.

		Execution time (s)		
Solver	Number of heaps	D4	D8	M
BARON	1	3.6	3.7	5.4
	2	11.4	23.7	3951.7
	3	9415	11494	120000
BONMIN	1	6.4	5.0	3.5
	2	81.7	95.6	8.1
	3	106.3	276.9	20.2
COUENNE	1	8.5	10.6	11.7
	2	39.8	31.3	78.7
	3	123	231	86271
SBB	1	3.0	2.9	2.3
	2	4.3	3.9	1.7
	3	4.2	5.3	1.6

Table 3. Parameters used in the performance analysis of MINLP solvers. NC: Does not converge.

		Profits (MUS$/y)								
		Copper price (kUS$/t)/Model								
		5.5			7.7			9.9		
Solver	Number of heaps	D4	D8	M	D4	D8	M	D4	D8	M
AlphaECP	1	NC	164	NC	344	NC	NC	NC	544	NC
	2	350	NC	NC	688	709	NC	NC	NC	NC
	3	NC	NC	NC	NC	NC	NC	NC	NC	NC
BARON	1	175	175	182	344	354	369	513	544	567
	2	350	350	363	688	709	738	1026	1089	1133
	3	525	525	545	1032	1063	1107	1540	1633	1700
BONMIN	1	175	175	182	344	354	369	513	544	567
	2	350	350	363	688	709	738	1026	1089	1133
	3	525	525	545	1032	1063	1107	1540	1633	1700
COUENNE	1	175	175	182	344	354	369	513	544	567
	2	350	350	363	688	709	738	1026	1089	1133
	3	525	525	545	1032	1063	1107	1540	1633	1700
DICOPT	1	175	175	182	344	354	369	513	544	567
	2	248	192	363	460	511	738	672	748	1133
	3	248	248	545	690	716	1107	1008	1061	1700
LINDOGlobal	1	175	175	182	344	354	369	513	544	567
	2	350	NC	363	688	NC	738	1026	NC	1133
	3	248	198	545	613	298	1107	1251	1121	1700
OQNLP	1	NC	NC	182	NC	NC	369	NC	NC	567
	2	NC	NC	NC	NC	NC	738	NC	NC	NC
	3	NC	NC	545	NC	NC	1107	NC	NC	NC
SBB	1	175	175	182	344	354	369	513	544	567
	2	350	350	363	688	709	738	1026	1089	1133
	3	525	525	545	1032	1063	1107	1540	1633	1700

three heaps, 101 equations, 96 variables and 3 binary variables were managed. Among the solvers that demonstrated convergence and optimum global values, SBB was the fastest MINLP solver, while BARON and COUENNE extremely increase the execution times. Although SBB is a local solver, it was selected to solve the following case studies based on its shown performance.

3.2. Study of cases

Previous models, as those proposed by Padilla et al. (2008) and Trujillo et al. (2014), analyzed the effect of design and operating variables on the economic optimization of heap leaching process. This study also analyses the optimization of design and operating variables, but in this case, focusing on the irrigation of leaching heaps and evaluating the economic benefits. A range of different irrigation rates was evaluated, between 0.008 and 0.012 m/h to determine the superficial velocity that will be used in the study of cases. Both the recovery to acid consumption are greater using larger superficial velocities, but the increment of recovery before 100 days is less significant than for acid consumption (Figure 2). Therefore larger profits can be achieved with large initial irrigation rates. According to this behavior, a lower and upper bound equal to 5 and 12 L/h/m^2 were defined for the irrigation rate. The other parameters used in the simulation of cases are listed in Table 2.

The effect of the variables: copper price, ore grade, variable cost and acid price were evaluated at the level values listed in Table 4, regarding the total recovery, cycle times and optimum configuration for each heap system, which includes the volumetric flow (m^3/cycle) of each stream. All of them were studied by using GAMS software and the SBB solver. This solver showed the best performances according to the search of the global optimum and execution time, discussed in the previous section. From now on, the Disjunctive model will employ eight disjunctions in the calculation process.

For the 2-heap system analyzed by the Mellado model, the optimum configuration and volumetric flows were the same for the copper price, acid price and variable cost, while for the ore grade, the flow values were different. The same behavior of the variables was observed for the 3-heap system. On the other hand, the Disjunctive model for the 2-heap system determined that copper price, variable cost, and acid price have the same optimum configuration and flows, while ore grade obtained another flowsheet. In the case of the 3-heap system, the results of process configuration and flows were different for each analyzed variable. The general configuration for the 2- and 3-heap system obtained by the Mellado and Disjunctive model is presented in Figure 3. The values of the streams are given in each subsection below. In Figure 3 (and Figure 5), H1, H2, H3, and SX symbolize the heap 1, 2, 3, and the solvent extraction unit. The name of the streams represents the origin and destination, for example, stream H1-H3 goes from heap 1 to heap 3.

To avoid an excess of information in the paper, the details about volumetric flows obtained from the analysis of parameters variable cost and acid price were put in the Appendix B, but relevant comments about them are mentioned in the corresponding subsection.

3.2.1. Copper price (P_{C_k})

The copper price has a direct incidence on the income, and for this analysis, 3 values were considered: 5.5, 7.7 and 9.9 kUS$/t. According to the simulations, for both models and all heap systems, an increase in the copper price results in a growth of the annual profits. Since the Disjunctive model is an approximation of the Mellado analytical model, their estimations are slightly less and the differences increase with the number of heaps (Table 6).

On the other hand, recoveries and cycle times were not affected by the variations of copper price; however, the total recoveries and cycle times increase and decrease with the number of heaps, respectively, as shown in Table 7.

The same optimum process configuration was reached by the different copper prices for each heap system (Figure 3a and Figure 3b). For the 2-heap system, the flow distribution varies slightly between the Mellado and Disjunctive models (Table 8). For 3-heap systems, the Mellado model obtained an

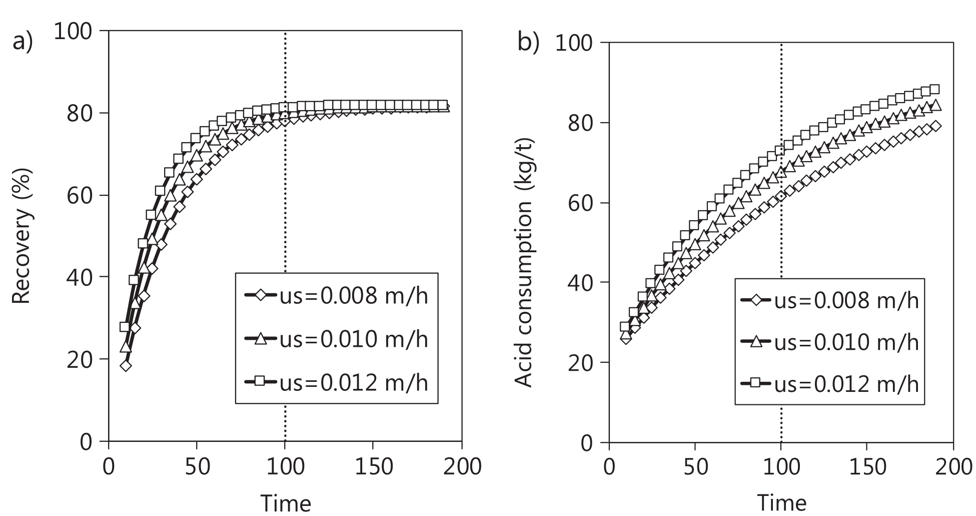

Figure 2. Simulated a) recovery and b) acid consumption at different irrigation rates along time by using Equations (A33) and (A37), respectively.

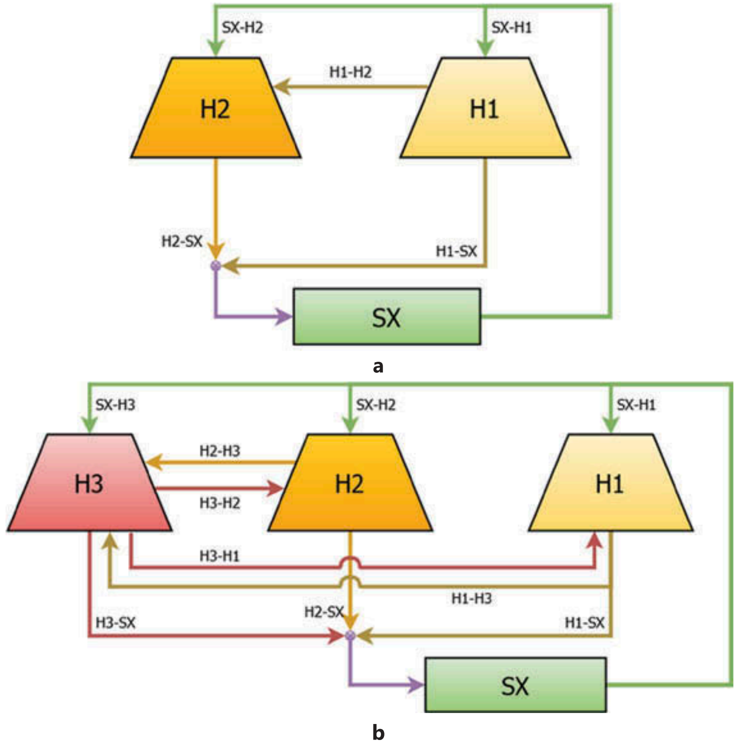

Figure 3. General structures of operations for the (a) 2-heap and (b) 3-heap systems obtained for the Disjunctive and Mellado models. H1, H2, H3, and SX denote the heap 1, 2, 3, and the solvent extraction unit. The streams are named considering the origin and destination of the streams.

Table 6. Profits obtained under different copper prices. Results calculated by the Disjunctive and Mellado models for different heap systems.

	Profit (MUS$/year)					
Copper price (kUS$/t)	1-heap		2-heap		3-heap	
	D8	M	D8	M	D8	M
5.5	518	530	821	839	1267	1285
7.7	887	904	1399	1426	2129	2185
9.9	1257	1278	1977	2013	3008	3084

Table 7. Cycle times and recoveries calculated for different heap systems by the Disjunctive and Mellado models. These results were also obtained by the simulations of variable cost (C_v) and acid price (P_A).

Number of heaps	Model	Cycle time (d)	Total recovery (%)
1	D8	43.5	69.7
	M	43.5	70.6
2	D8	30.5	76.5
	M	30.1	76.7
3	D8	20.0	76.3
	M	19.5	76.3

optimum structure that reaches the same values for either copper prices

The Disjunctive model exposed 3 different structures that were specific for each copper price. In all of the cases comprising 3 heaps, the solution obtained from the first heap, which is the least concentrated among the heap outlets, is conduced to the last heap to concentrate the solution, and a small fraction of the H1 PLS is directly sent to SX unit. In this sense, H2 and H3 are the most important feeding streams to the SX unit, independent of the copper price. At the extent that the copper price increases, the inventory of intermediary

Table 8. Optimum volumetric flows obtained under different copper prices, for 2- and 3-heap systems by the Disjunctive and Mellado models.

	Volumetric flow (10^5 m^3/cycle)					
	2-heap		3-heap			
Stream	D8	M	D8 (P_{C_k}=5.5)	D8 (P_{C_k}=7.7)	D8 (P_{C_k}=9.9)	M
H1-SX	7.5	7.7	2.5	3.3	2.3	2.5
H1-H3			8.8	8.3	9.3	8.7
H1-H2	10.1	9.6				
H2-SX	17.5	17.3	11.3	10.2	11.5	11.2
H2-H3				1.3		-
H3-SX			11.3	11.5	11.2	11.2
H3-H1					0.3	
SX-H1	17.5	17.3	11.3	11.5	11.2	11.2
SX-H2	7.5	7.7	11.3	11.5	11.5	11.2
SX-H3			2.5	2.0	2.3	2.5

solutions also increases (apart from the H1-H3 flow that is present in all cases), appearing the flows H2 to H3 and H3 to H1. This condition is produced to achieve larger recoveries that increase the incomes.

3.2.2. *Variable cost* (C_v)

The variable costs considered in the analysis were: 0.005, 0.010 and 0.020 kUS$/t, and are related to costs that vary in proportion to the production, such as water and energy consumption, ore transport and maintaining costs. The cycle times and recoveries were the same than obtained in the simulation of copper price (Table 7). As the variable costs have direct incidence on the expenses, the profits are also affected. As shown in Table 9, to the extent that increases the amount of heaps, the benefits are greater, and the values

Table 9. Profits obtained under different variable costs. Results calculated by the Disjunctive and Mellado models for different heap systems.

Variable cost (kUS$/t)	Profit (MUS$/year)					
	1-heap		2-heap		3-heap	
	D8	M	D8	M	D8	M
0.005	994.4	1010.8	1551.9	1580.5	2361.8	2422.5
0.010	860.6	877.0	1361.1	1387.1	2071.1	2124.9
0.020	593.0	609.3	949.5	1005.5	1489.7	1578.2

Table 10. Profits obtained under different acid prices. Results calculated by the Disjunctive and Mellado models for different heap systems.

	1-heap		2-heap		3-heap	
	D8	M	D8	M	D8	M
Acid price (kUS$/t)	*Profit (MUS$/year)*					
0.110	930.7	946.9	1474.3	1501.2	2242.6	2298.7
0.162	887.4	903.8	1399.2	1425.8	2129.2	2184.5
0.210	847.4	863.9	1329.9	1356.2	2024.5	2079

obtained by the Disjunctive model were slightly lower than those calculated by the Mellado model.

The configuration and flow distribution for the 2-heap system was identical as for the copper price analysis by both models, and in the case of the 3-heap system, the solving of Mellado model was equally the same than the previous analysis. However, the results of the simulations by the Disjunctive model for the 3-heap system achieved slight differences in the stream configuration, which was different for each evaluated variable cost. The contribution of inventory of solution to the SX unit from H3 are kept relatively constant while increasing C_v, the flow from H1 becomes greater (about 4 times) and disappearing the volumetric flow of the stream H2-SX. Thus, working with high variable costs, H2 operates as intermediary heap to concentrate the solutions and conducing them to H3. H2 does not exchange flows (input or output) with the SX unit.

3.2.3. Acid price (P_A)

The acid prices considered were: 0.110, 0.162 and 0.210 kUS $/t. The use of sulphuric acid is closely related to the copper mining, and its price will depend on the global demand for its application in the hydrometallurgical processes. Hence this variable was studied separately of the operating costs. The cycle times and recoveries were the same than obtained in the simulation of copper price and variable cost (Table 7). To the extent that the number of heaps increases, also increase the profits, although if the acid is purchased more expensively, these profits decrease (Table 10).

The structure and flow distribution that resulted from the 2-heap system was the same to that obtained in the copper price and variable cost analysis by both models. Moreover, the simulation reached by Mellado model for the 3-heap system was also the same. All of them are listed in Table 8. Regarding the Disjunctive model performance for the 3-heap system, the distribution of the streams and their volumetric flows were slightly different from ones obtained under different copper price scenarios. The feeding of SX is practically dominated by the concentrated streams that come from H2 and H3 heaps. Increasing the acid price results in that a greater fraction of the pregnant solution of H1 is led to H3. Independently of the acid price, H1 is basically leached with the solution coming from the SX unit, trying to re-use as much as possible acid and reduce its replenishing rate. The other streams remained similar than ones obtained in the analysis of P_{C_k} y C_v.

3.2.4. Ore grade

The optimization was performed considering ore grades of 0.5, 0.9, and 1.3%. Better economic benefits will be achieved if the processed minerals have higher grades. In this case, the total recovery levels are independent of the ore grade, but the profits and cycle time are directly influenced by the copper composition (Table 11). The cycle time decreases when more heaps are managed, operating for a longer time the first heap(H1) and then diminishing in the subsequent heaps. The variations of times are proportional to the ore grade for systems of more than one heap. For this case, the models select constant cycle times.

Due to the variation of recovery in each heap is very small between the models, and to simplify the presentation of the results, Figure 4 shows only the values obtained by the Mellado model, but the deviation between models is less than 3%. The notation at the base of bars corresponds to a specific heap of total heaps composing the system, e.g.: 1/2H represents the first heap of the 2-heap system. The disjunctive model in all cases slightly underestimates the results of the Mellado model. In the same way as cycle time, for the 3-heap system, the copper recovery of H1 was the highest while for the other heaps were lower. The recovery of each heap is reduced since shorter cycle times were used in subsequent heaps. In all the scenarios, global recoveries larger than 70% were achieved with similar results between 2- and 3-heaps and slightly less by the 1-heap system. The influence of the ore grade is mainly reflected in the stream flow distribution when more than a heap is operated.

Unlike the previous cases for the 2-heap system, where both models achieved a unique structure with the same volumetric flows, in this case, the volumetric flows were specific for each ore grade scenario (Figure 5). In the same way, for the 3-heap system, the models obtained specific structures with unequal flows for each ore grade levels (Table 12). The calculation of the volumetric flows appears to be strongly sensitive to the ore grade variations, because this parameter affects the processing performance while the other tested parameters (copper and acid price and variable costs) are related to the economics of the process.

The stream H2-H1 was not chosen by any model. It may be since the system determines that the solution can be concentrated more and brings the stream to a less washed heap, i.e. H3.

Table 11. Cycle times and profits obtained under different ore grades. Results calculated by the Disjunctive and Mellado models for different heap systems.

	1-heap		2-heap		3-heap	
	D8	M	D8	M	D8	M
Ore grade (%)	*Cycle time (d)*					
0.5	43.5	43.5	31.2	32.0	20.3	22.1
0.9	43.5	43.5	30.5	30.1	20.0	19.5
1.3	43.5	43.5	30.2	29.8	20.0	19.2
	Profit (MUS$/year)					
0.5	288.0	296.8	450.2	462.9	691.3	704.6
0.9	887.4	903.8	1399.2	1425.8	2129.2	2184.5
1.3	1486.8	1510.7	2359	2399.2	3555.4	3689.2

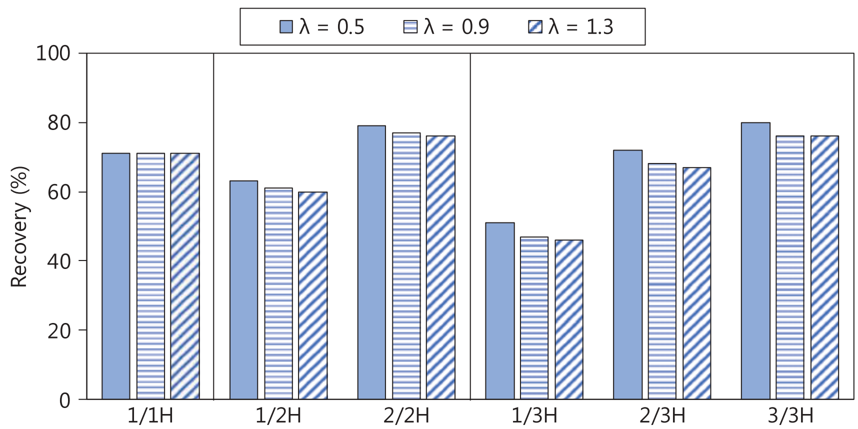

Figure 4. Simulated recoveries by the Mellado model for different ore grades (0.5, 0.9, 1.3%) and heap systems comprised by one (1H), two (2H) and three (3H) heaps. Each heap that composes the system is individualized (the number before the slash). Results of the Disjunctive model reached similar values.

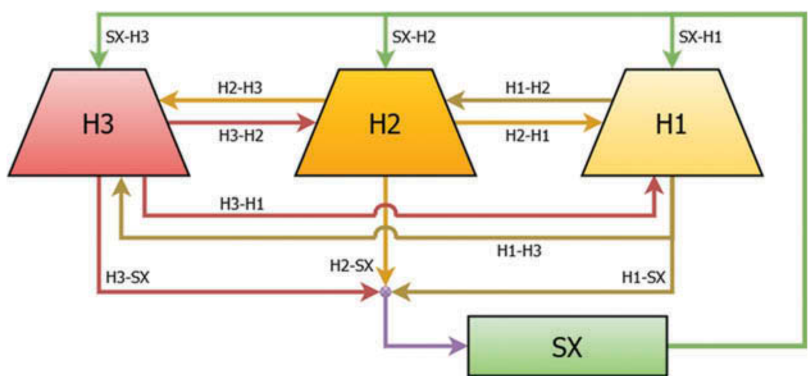

Figure 5. General structure of the heap leaching system obtained for the ore grade analysis by the Disjunctive and Mellado model. H1, H2, H3, and SX denote the heap 1, 2, 3, and the solvent extraction unit respectively. The streams are named considering the origin and destination of the streams.

Table 12. Optimum volumetric flows obtained under different ore grades, for 2- and 3-heap systems by the Disjunctive and Mellado models.

	Volumetric flow (10^5 m^3/cycle)											
	2-heap						3-heap					
Stream	D8 (λ_k=0.5)	D8 (λ_k=0.9)	D8 (λ_k=1.3)	M (λ_k=0.5)	M (λ_k=0.9)	M (λ_k=1.3)	D8 (λ_k=0.5)	D8 (λ_k=0.9)	D8 (λ_k=1.3)	M (λ_k=0.5)	M (λ_k=0.9)	M (λ_k=1.3)
H1-SX	7.0	7.5	7.7	7.7	7.7	7.9	1.6	3.3	3.8	2.3	2.5	2.9
H1-H3							10.1	8.3	7.8	10.4	8.7	8.2
H1-H2	10.9	10.1	9.7	10.8	9.6	9.3						
H2-SX	18.0	17.5	17.4	17.4	17.3	17.1	11.7	10.2	11.5	11.2	11.2	11.1
H2-H3								1.3				
H3-SX							11.7	11.5	9.7	11.5	11.2	11.1
H3-H1									1.8			
SX-H1	18.0	17.5	17.4	18.4	17.3	17.1	11.7	11.5	9.7	12.7	11.2	11.1
SX-H2	7.0	7.5	7.7	6.6	7.7	7.9	11.7	11.5	11.5	11.1	11.2	11.1
SX-H3							1.6	2.0	3.8	11.5	2.5	2.9

4. Conclusions

The developed methodology allowed to design and plan leaching processes by a superstructure of heap systems and a set of equations that facilitate the prediction of recoveries, cycle and leaching times, economic benefits and volumetric flows, among others. For the resolution of the mathematical algorithms, it was determined that the solvers BONMIN, BARON, COUENNE and SBB reach the global optimum, but with different execution times, where BARON was the most time-consuming solver, about 5000 times more than SBB, which was the solver that required the least time.

Leaching times and recovery are affected by the ore grade, the superficial velocity and the concentration of the input to the SX unit. Faster recoveries are shown by heaps irrigated with higher flows; however, an intensive irrigation results in more diluted solutions that results in longer periods of time in each cycle to obtain high recoveries. For systems comprised by more than a heap, leaching time of each unit decreases, longer operating in the first heap and decreasing consecutively in the following heaps, so cycle times are shortened with a larger number of heaps.

The variables: copper price, acid price, variable cost and ore grade were studied to determine the influence on the distribution of volumetric flows. The first three are parameters that affect the incomes and outcomes, but not the operational management. On the other side, ore grade is involved in the technical processing. Results show that the process configuration is unaffected by the economic parameters for simple systems, while the ore grade changes the distribution of volumetric flows of the streams.

In general, the solution obtained try to reach the maximum concentration of copper previous to SX unit, for this, the efforts of recycling flows are focused on the first heaps and some possible streams were not chosen by both models. In all the analyzed variables, the results for both models were very similar, resulting in slightly higher values by the model Mellado. It was demonstrated by both models, that as the number of heaps increases, more metal is recovered, and thus the incomes are significantly higher than for a single heap system. Finally, this study validated the influence of the heap height, leaching time, and irrigation rate (superficial velocity) in the leaching process, under an optimization performed from an economic perspective.

5. Nomenclature

5.1. Sets

$J = \{(j)/(j), process\ unit\}$

$JP = \{(jp)/(jp \in J), heap\ leaching\ unit\}$

$SX = \{(sx)/(sx \in J), solvent\ extractionunit\}$

$K = \{(k)/k \in K), leachable\ species\}$

$D = \{(d)/(d \in D), disjuntions\ for\ recovery\ and\ acid\ consumption\}$

5.2. Variables y parámetros

A	heap area [m^2]
a_d,b_d	recovery constants for the Disjunctive model
a^c_d,b^c_d	acid consumption constants for the Disjunctive model
C	costs [MUS$]
$C_{T_{jp}}$	total cost[(kUS$)⁄(m^2 cycle)]
$C_{V_{jp}}$	variable cost [(kUS$)⁄(t ore)]
$C^{\infty}_{H^+}$	acid consumption at infinite time [kg/t]
$C_{H^+_{jp}}$	acid consumption of heap jp [kg/t]
$C^0_{H^+}$	acid consumption at the beginning [kg/t]
$C_{A_{jp}}$	acid consumption of heap jp [kg/cycle]
C_{A_T}	total acid consumption [kg/cycle]
$C_{(pre-leac)_{jp}}$	operational costs before leaching [(kUS$)⁄(t ore)]
$C_{post-leac}$	operational costs after leaching [(kUS$)⁄(t Cu)]
$D_{jp,k}$	linear availability of $k$species in the heap jp [(t Cu)⁄m]
H	planning time horizon [d]
I	incomes[MUS$]
k_1,k_2	kinetic constants
$L_{l,m,k}$	mass of the unit process l *to* m *of the* k species [t⁄cycle]
$L_{i,j,k}$	mass of the unit process i *to* j *of the* k species [t⁄cycle]
$L^{in}_{j,k}$	input mass of the unit process j *of the* k species [t⁄cycle]
$L^{out}_{j,k}$	output mass of the unit process j *of the* $k$species [t⁄cycle]
$M_{jp,k}$	mass of mineral loaded on heap jp *of* k species [t]
M	constant of in the big M method
$MW_{H_2SO_4}$	molecular weight of sulphuric acid [t⁄tmol]
MW_k	molecular weight of k species [t⁄tmol]
N	number of cycles
P_k	production of k species [t⁄cycle]
P_{C_k}	price of k species [kUS$/t]
P_A	price of acid [kUS$/t]
Q_i	volumetric flow of the process unit i [m^3⁄cycle]
$QI_{i,j}$	volumetric flow of the process unit i *to* j [m^3⁄cycle]
Q_0	volumetric flow at the SX unit outlet [m^3⁄cycle]
Qr_{jp}	irrigation rate [m^3⁄d]
$R_{jp,k}$	recovery from heap jp*of the* k species [%]
$R_{jp,k,d}$	recovery from heap jp *of the* k species in the disjunctive model [%]
R_∞	recovery at infinite time [%]
t	cycle time of the heap system [d]
t_{jp}	end time of leaching of heap jp [d]
us_{jp}	superficial velocity [cm⁄s]
U	profits [MUS$]
$x_{0,k}$	concentration of the output stream from SX unit of k species [t⁄m^3]
$x_{j,i,k}$	concentration of the unit process jp *to* i *of the* k species [t⁄m^3]
$x^{in}_{j,k}$	concentration of the input stream of the unit process j *of the* k species [t⁄m^3]
$x^{out}_{j,k}$	concentration of the output stream of the unit process j of the k species [t⁄m^3]
y_{jp}	binary variable of the cycle times
y_d	disjunctive binary variable
$y2_{jp,d}$	disjunctive binary variable of the analyzed heap
$y3_{ip,jp}$	selection of the disjunction interval of the analyzed heap
z	heap height [m]

5.3. Greek letters

$\alpha, \beta, \lambda, \omega$	recovery constants of the Mellado model
α_H, β_H	acid consumption constants
ρ	ore density [t⁄m^3]
λ_k k	species grade in the ore [%]

5.4. Superscripts

LO	lowerbound
UP	upperbound

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Appendix A Mathematical model

Mathematical model of the proposed methodology. The equations are developed considering the Figure 1, which represents the set of alternative flows that are distributed through the heaps. A heap leaching unit *jp* is represented by the structure in Figure 1. *L* represents the mass flow and the subscripts *i* and *j* denote processing units. The general set of units and symbols used along the text are defined in the nomenclature section. A mixer at the entrance of the processing units and a divider at the output are considered. Heaps are arranged in series and flows between them and to a solvent extraction unit are allowed, but a stream output cannot enter to the same unit again. The solvent extraction unit receives the solution that comes from each heap its output feeds the heaps.

In the heap unit *jp*, Equation(A1) shows the mass balance conducted for the *k* species, depending on the mass flows in and out.

$$L_{jp,k}^{out} = L_{jp,k}^{in} + M_{jp,k}\left(R_{jp,k} - R_{jp-1,k}\right)\forall jp \in JP \quad (A1)$$

Where $L_{jp,k}^{out}$ and $L_{jp,k}^{in}$represent the mass of output and input of the k species in the heap jp, respectively, and $M_{jp,k}$ refers to the mass amount of loaded ore in the heap leaching units that is determined by Equation (A2), where the ore grade (λ_k) density (ρ), the heap height (z) and heap area (A) are considered.

$$M_{jp,k} = \lambda_k \rho z A \forall jp, \in JP \quad (A2)$$

On the other hand, $R_{jp,k}$, is the recovery of the k species, which is given in terms of design and operating variables and used in Equations (A32) and (A33). The term $(R_{jp,k} - R_{jp-1,k})$ is included in Equation(A1) since heaps are operated in series. In the balance of equipment, Equation (A3) shows the mass balance of the mixers and Equation(A4) the balance of the dividers, depending on the corresponding flows. $L_{j,k}^{in}$is the mass input to the j processing unit of the k species (t/cycle) and $L_{j,k}^{out}$ is the mass output of the j processing unit of the k species.

$$L_{j,k}^{in} = \sum_{\substack{i \in J \\ j \neq i}} L_{i,j,k} \quad \forall j \in J, k \in K \quad (A3)$$

$$L_{j,k}^{out} = \sum_{\substack{i \in J \\ j \neq i}} L_{j,i,k} \quad \forall j \in J, k \in K \quad (A4)$$

For solvent extraction unit, a mass balance was performed according to the kind of production value (P_k), given by the Equation(A5).

$$P_k = \sum_{sx \in SX} L_{sx,k}^{in} - L_{sx,k}^{out} \quad (A5)$$

Where $L_{sx,k}^{in}$ is the input mass that enters into the SX unit of the k species and $L_{sx,k}^{out}$ corresponds to the output mass of the same unit. A time horizon was defined as the period during which the process will be evaluated, from which the number of cycles (N), or number of times that the process is repeated on the time horizon, H (Equation(A6)).

$$Nt = H \quad (A6)$$

The cycle time is defined as the total amount of time required to complete the process and is given by the following equation:

$$t = \max_j\left(t_{jp} - t_{jp-1}\right) \quad (A7)$$

To represent t as a disjunctive expression, it results:

$$\bigvee_{jp \in JP} \begin{bmatrix} y_{jp} \\ t = t_{jp} - t_{jp-1} \\ t_{jp} - t_{jp-1} > t_{ip} - t_{ip-1} \quad \forall ip \in JP, ip \neq jp \end{bmatrix} \quad (A8)$$

Where y_{jp} is a binary variable that corresponds to the selection of cycle time in days and t_{jp} is the leaching time of the heap jp. The method of the Big M was applied for the disjunctive expressions for cycle time (Biegler et al., 1997). For this, a letter M was defined as a very large but finite number, which is used as the coefficient of artificial variables in the function.

$$t_{jp} - t_{jp-1} \geq \left(t_{ip} - t_{ip-1}\right) - M\left(1 - y_{jp}\right) \quad \forall jp \in JP, \forall ip \in JP, \forall ip \neq jp \quad (A9)$$

$$\sum_{jp \in JP} y_{jp} = 1 \quad (A10)$$

$$t \leq t_{jp} - t_{jp-1} + M\left(1 - y_{jp}\right) \quad \forall jp \in JP \quad (A11)$$

$$t \geq t_{jp} - t_{jp-1} - M\left(1 - y_{jp}\right) \quad \forall jp \in JP \quad (A12)$$

In Equation (A10), y_{jp} represents a single working range, and Equations (A11) and (A12) correspond to lower and higher time cycle equations, respectively. Note that these equations are activated when y_{jp} is equal to one. Other conditions regarding the operating parameters in function of leaching time (t_{jp}) that have to be fulfilled are determined by the Equations from (A13) to (A21).

$$0 < t_{jp} < t^{UP} \quad \forall jp \in JP \quad (A13)$$

$$0 < t_{jp} - t_{jp-1} \leq t^{UP} \quad \forall jp \in JP \quad (A14)$$

$$\frac{H}{t^{UP}} < N < \frac{H}{t^{LO}} \quad (A15)$$

$$t^{LO} < t < t^{UP} \quad (A16)$$

$$L_{j,i,k}^{LO} \leq L_{j,i,k} \leq L_{j,i,k}^{UP} \quad \forall j, i \in J, j \neq i \quad (A17)$$

$$0 < R_{jp,k} < R_{\infty} \quad \forall j \in J \quad (A18)$$

$$z^{LO} < z < z^{UP} \quad (A19)$$

$$Qr_{jp}^{LO} < Qr_{jp} < Qr_{jp}^{UP} \quad (A20)$$

$$us_{jp}^{LO} < us_{jp} < us_{jp}^{UP} \quad (A21)$$

The irrigation rate Qr_{jp} is determined by the relationship between the volumetric flow in the unit jp (Q_{jp}) and the cycle time, shown in Equation(A22). Input and output parameters are determined in Equations (A23) and (A24).

$$Qr_{jp} = \frac{Q_{jp}}{t} \quad (A22)$$

$$Q_i = \sum_{\substack{i \in J \\ j \neq i}} QI_{i,j} \quad \forall j \in J \quad (A23)$$

$$Q_i = \sum_{\substack{i \in J \\ j \neq i}} QI_{j,i} \quad \forall j \in J \quad (A24)$$

In the above equations, Q_i represents the volumetric flow of the processing unit, which is the sum of the streams that flows from the unit $i$to j. Note that Equations (A23) and (A24) assume that the density is constant, and for this, the input and output volumetric flowrates are equals.

In the case of a homogeneous heap that is uniformly irrigated, the irrigation rate is closely related to the superficial velocity (us_{jp}). In the proposed model, the superficial velocity is included as operational variable, which can be determined by Equation(A25).

$$us_{jp} = \frac{Qr_{jp}}{A} \quad (A25)$$

Each stream is determined by the relationship between the flow and concentration, given by:

$$L_{i,j,k} = x_{i,j,k} QI_{i,j} \quad (A26)$$

Where $x_{i,j,k}$ is the concentration of k-species in the stream that runs from the processing unit i to j and $QI_{i,j}$ is the volumetric flow from the processing unit $i$to j. The inflows and outflows are determined in Equations(A27) and (A28), where $L_{i,k}^{in}$ and $L_{i,k}^{out}$ are the mass flowrates of the k species in the processing unit i for the input and output streams, respectively. Both are obtained by multiplying the corresponding concentration of the volumetric flow with the processing unit i, as shown below:

$$L_{i,k}^{in} = x_{i,k}^{in} \cdot Q_i \quad (A27)$$

$$L_{i,k}^{out} = x_{i,k}^{out} \cdot Q_i \quad (A28)$$

The maximum flow for the k-species, $L_{j,i,k}^{UP}$, is determined by

$$L_{j,i,k}^{UP} = x_{j,i,k}^{UP} \cdot Q_{j,i}^{UP} \quad (A29)$$

Where $x_{j,i,k}^{UP}$ represents the maximum concentration of the k species of the processing unit j to i and $Q_{j,i}^{UP}$ is the maximum volumetric flow from the unit $j$to i. It should be considered that the concentration in the outlet stream $x_{i,k}^{out}$, shall be equal to the concentration of the stream of processing unit i, as follows:

$$x_{i,k}^{out} = x_{i,j,k} \forall jp \in JP, \forall ip \in JP, \forall ip \neq jp \tag{A30}$$

In this work, the copper recovery will be calculated by two models: a) an analytical model proposed by Mellado et al. (2011a, 2009), where the recovery was analyzed in function of design and operating variables; b) through disjunctive functions utilizing interval sections of time to approximate the predicted recoveries obtained from the Mellado model. This model is based on the constitutive equations in function of design and operational variables (Equation(A31)), where z is the heap height, us is the superficial velocity of irrigation, r is the particle radius, and$t$ is the leaching time.

$$R_{jp,k} = f(z, us, r, t) \ \forall jp \in JP \tag{A31}$$

In this work, the heap height was firstly analyzed, through Equation (A32), where R_∞ represents the recovery at infinite time and the coefficients α and $\beta$are adjustable recovery constants.

$$R_{jp,k} = R_\infty \left(1 - e^{-\alpha\left(t_{jp} - \beta z\right)}\right) \ \forall jp \in JP, k \in K \tag{A32}$$

The effect of the irrigation on the leaching performance was also studied, using the following expression (Equation(A33)), which considers others recovery ($\lambda$and ω) and kinetic constants (k_1 and k_2).

$$R_{jp,k} = R_\infty \left[1 - \lambda e^{-k_1\left(us_{jp} t_{jp} - \omega\right)} - (1-\lambda) e^{-k_2\left(t_{jp} - \frac{1}{us_{jp}}\omega\right)}\right] \ \forall jp \in JP, k \in K \tag{A33}$$

The velocity at which the flows are moved will be optimized, complying the following restriction:

$$us_{jp} \leq \frac{tp_{jp}}{\omega} \tag{A34}$$

Table A1. Parameters used by the Mellado analytical model for the copper recovery calculation (Mellado et al., 2011b, 2009).

Parameter	Value
Recovery at infinite time, R^∞	81.8
Recovery constant, α	0.02
Recovery constant, β	1
Recovery constant, λ	0.81
Recovery constant, ω	0.002
Kinetic constant, k_1	141.4
Kinetic constant, k_2	37.4

The parameter values used in Equations(A32) and (A33)are detailed in Table A1.

For the calculation of the acid consumption by Mellado model, the general equation is defined in function of the design and operational variables:

$$C_{H_{jp}}^{+} = f(z, us, r, t) \ \forall jp \in JP \tag{A35}$$

When the effect of the heap height is studied the acid consumption is determined by Equation(A36), meanwhile when the effect of the irrigation rate is analyzed, Equation(A37) is used.

$$C_{H_{jp}}^{+} = C_{H^+}^{\infty}\left(1 - e^{-\alpha_{H^+}\left(t_{jp} - \beta_{H^+} z\right)}\right) + C_{H^+}^{0} \tag{A36}$$

$$C_{H_{jp}}^{+} = C_{H^+}^{\infty}\left[1 - \lambda_{H^+} e^{-k_{1_{H^+}}\left(us_{jp} t_{jp} - \omega\right)}\right] + C_{H^+}^{0} \ \ \forall jp \in JP \tag{A37}$$

$C_{H^+}^0$represents the initial acid consumption, $C_{H^+}^\infty$ is the consumption at the infinite time, i.e. the maximum quantity of acid that can be consumed, α_{H^+}, β_{H^+}, λ_{H^+} and $k_{1_{H^+}}$ are acid consumption coefficients. The values of these coefficients are listed in Table A2.

Table A2. Parameters used by the Mellado model for the acid consumption calculation (Mellado et al., 2011b, 2009).

Parameter	Value
Acid consumption at infinite time, $C_{H^+}^\infty$ (kg/t)	71.0
Initial acid consumption, $C_{H^+}^0$ (kg/t)	25
Acid consumption constant, α_{H^+}	0.013
Acid consumption constant, β_{H^+}	1
Acid consumption constant, λ_{H^+}	1
Kinetic constant, $k_{1_{H^+}}$	33.7

For the disjunctive model, the recovery and acid consumption is expressed by:

$$\bigvee_{d \in D} \begin{bmatrix} y_{jp,d} \\ R_{jp,k} = a_d + b_d t_{jp} \\ C_{H_{jp}}^{+} = a_d^c + b_d^c t_{jp} \\ z = z_d \\ us_{jp} = us_{jp,d} \\ t_d^{LO} < t_{jp} < t_d^{UP} \end{bmatrix}, \forall jp \in JP, k \in K \tag{A38}$$

Where a_d, b_d, a_d^c and b_d^c are constants used in the recovery and acid consumption approximation and y_d is a binary variable that represents each disjunction. t_d^{LO}andt_d^{UP} are the lower and upper bound of leaching time of each disjunction, and us_{jp_d} is the superficial velocity in each disjunction. The convex hull method (Biegler et al., 1997) was used to transform Equation(A38) in a set of linear equations, according to:

$$R_{jp,k,d} = a_d y_{jp,d} + b_d t_{jp,d} \quad \forall jp \in JP, k \in K \tag{A39}$$

$$R_{jp,k} = \sum_d R_{jp,k,d} \quad \forall jp \in JP, k \in K \tag{A40}$$

$$C_{H_{jp,d}}^{+} = a_d^c y_{jp,d} + b_d^c t_{jp,d} \quad \forall jp \in JP, k \in K \tag{A41}$$

$$C_{H_{jp}}^{+} = \sum_d C_{H_{jp,d}}^{+} y_{jp,d} \quad \forall jp \in JP, k \in K \tag{A42}$$

$$t_{jp} = \sum_d t_{jp,d} \quad \forall jp \in JP \tag{A43}$$

$$y_{jp,d} t_d^{LO} < t_{jp,d} < t_d^{UP} y_{jp,d} \quad \forall jp \in JP, d \in D \tag{A44}$$

$$us_{jp} = \sum_d us_{jp_d} y_{jp,d} \quad \forall jp \in JP \tag{A45}$$

$$z = \sum_d z_d y_{jp,d} \tag{A46}$$

$$\sum_d y_{jp,d} = 1 \quad \forall jp \in JP \tag{A47}$$

The sulphuric acid constitutes a very important factor in the cost of the copper leaching, for this reason, its consumption (Equation(A48)) is included in the model due to it influences on the economic optimization of the process.

$$C_{A_{jp}} = \left(C_{H_{jp}}^{+} - C_{H_{jp-1}}^{+}\right)\rho A z - MW_{H_2SO_4} \sum \frac{u_k\left(R_{jp,k} - R_{jp-1,k}\right)}{MW_k} M_{jp,k}, \forall j \in J \tag{A48}$$

$C_{H_{jp}}^{+}$ represents the acid consumption in the heap jp, $C_{H_{jp-1}}^{+}$ is the consumption of the previous heap, $MW_{H_2SO_4}$ is the molecular weight of acid, u_k is the stoichiometry reaction coefficient equals to 2, MW_k is the molecular weight of the k species and $M_{jp,k}$ corresponds to the mass of k species in the ore loaded in the heap jp. The total acid consumption is the sum of the acid consumption of each heap, i.e.:

$$C_{A_T} = \sum_{jp} C_{A_{jp}} \tag{A49}$$

Appendix B

Optimum stream configurations and volumetric flows resulted by the simulation of the Disjunctive and Mellado models for the analysis of variable cost (Table B1) and acid price (Table B2).

Table B1. Optimum volumetric flows obtained under different variable costs, for 2- and 3-heap systems by the Disjunctive and Mellado models.

	Volumetric flow (10^5 m^3/cycle)					
	2-heap		3-heap			
Stream	D8	M	D8 (C_v=0.005)	D8 (C_v=0.010)	D8 (C_v=0.020)	M
H1-SX	7.5	7.7	3.3	3.9	14.9	2.5
H1-H3			8.3	7.6	2.3	8.7
H1-H2	10.1	9.6	–	–	–	–
H2-SX	17.5	17.3	10.2	11.5	–	11.2
H2-H3			1.3	–	7.2	–
H3-SX			11.5	9.6	10.1	11.2
H3-H1			–	1.9	–	–
H3-H2			–	–	7.2	–
SX-H1	17.5	17.3	11.5	9.6	17.3	11.2
SX-H2	7.5	7.7	11.5	11.5	–	11.2
SX-H3			2.0	3.9	7.7	2.5

Table B2. Optimum volumetric flows obtained under different acid prices, for 2- and 3-heap systems by the Disjunctive and Mellado models.

	Volumetric flow (10^5 m^3/cycle)					
	2-heap		3-heap			
Stream	D8	M	D8 (P_A=0.110)	D8 (P_A=0.162)	D8 (P_A=0.210)	M
H1-SX	7.5	7.7	3.9	3.3	2.0	2.5
H1-H3			7.6	8.3	9.5	8.7
H1-H2	10.1	9.6	–	–	–	–
H2-SX	17.5	17.3	11.5	10.2	11.5	11.2
H2-H3			–	1.3	–	–
H3-SX			9.6	11.5	11.5	11.2
H3-H1			1.9	–	–	–
SX-H1	17.5	17.3	9.6	11.5	11.5	11.2
SX-H2	7.5	7.7	11.5	11.5	11.5	11.2
SX-H3			3.9	2.0	2.0	2.5